On the Self Potential (SP) Interpretation for streaming potential characterization: A short review

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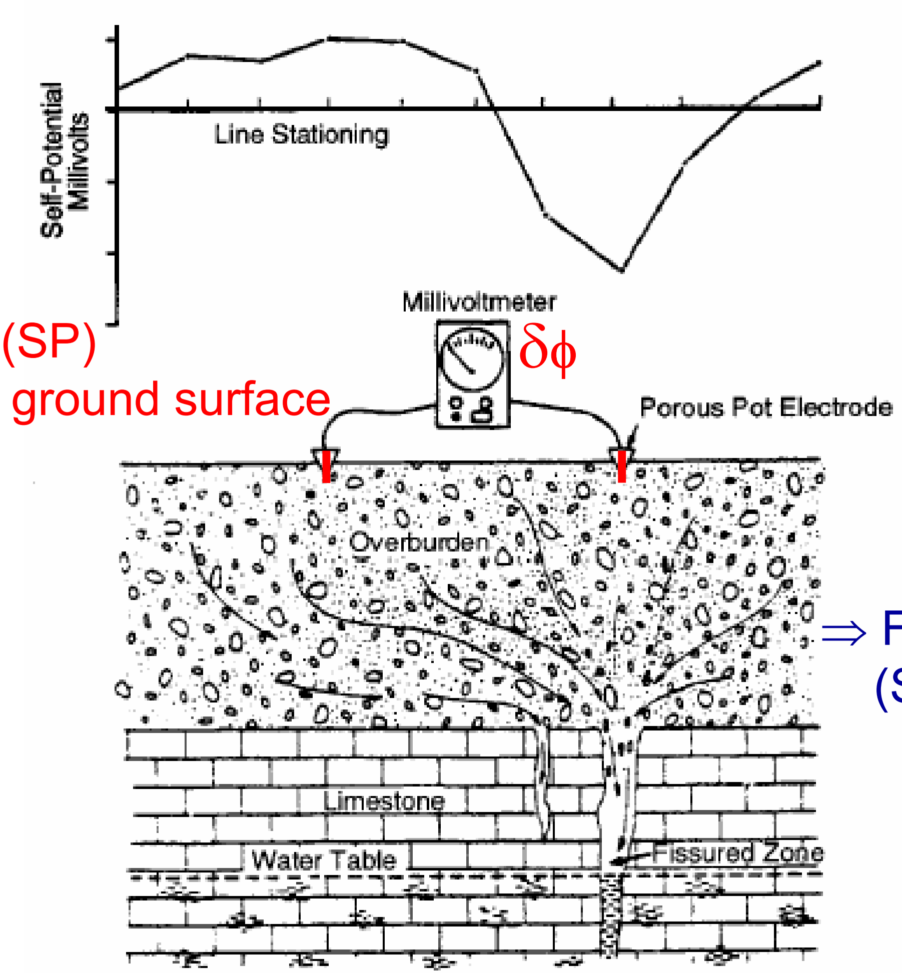
EAGE-Near Surface, Palermo, Italy, Sept., 2005,
Workshop on Hydrogeophysics

Objectives: Find information on fluid flows based upon interpretation of surface SP data

Self-Potential data (SP) measured near the ground surface



⇒ Fluid circulations in depth (SP sources)

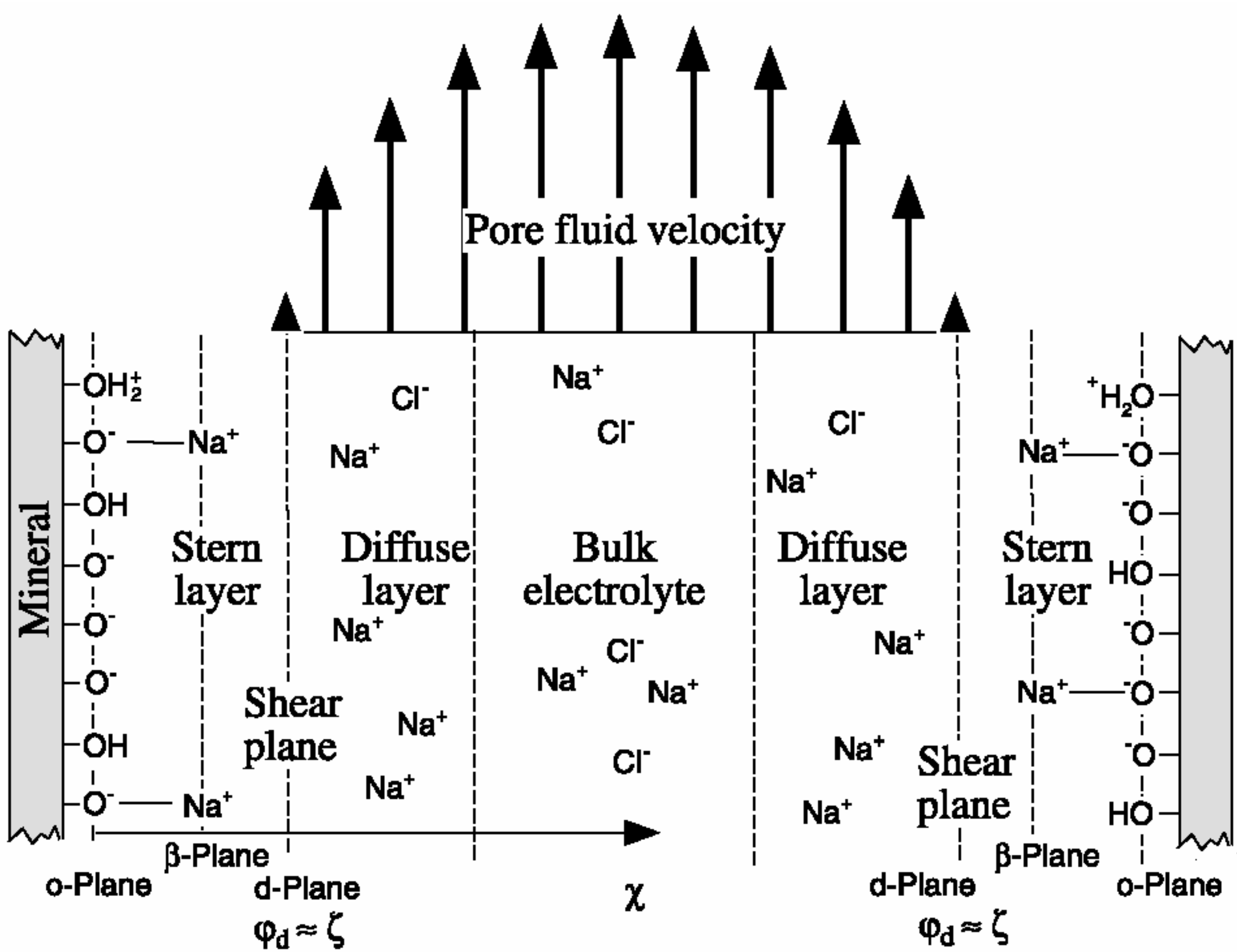
(Fig. from Ogilvy and Bogoslovsky, 1979)

Origins

Fundamentals:
- Helmholtz, Wiss. Abhandl. physic. tech. Reichsantalst I, 1879.
- Smoluchowski, Physikalische Zeitschrift, 1905.
- Gouy, J. Phys. radium, 1910.
- Chapman, Phil. Mag., 1913.
- Debye and Hückel, Physikalische Zeitschrift, 1923.
- Stern, Z. Electrokem., 1924.
- Onsager, Phys. Rev., 1931.
- Overbeek, *Colloid Science*, Elsevier, 1960.

Early applications in hydrogeophysics:
- Ogilvy, Ayed, and Bogoslovsky, Geophys. Prospect., 1969.
- Abaza and Clyde, Water Res. Research., 1969.
- Fitterman, J. Geophys. Res., 1978.
- Corwin and Hoover, Geophysics, 1979.
- Ishido and Mizutani, J. Geophys. Res., 1981.

Principle from the pore scale: Excess charge in the Diffuse layer



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(Fig. from Revil, Naudet, Nouzaret and Pessel, 2003)

Principle from the macroscopic scale: electrokinetic coupling

When electricity and hydraulics run separately:

	Hydraulic modelling	Electric modelling
Flux	$\vec{q} = -K\vec{\nabla} h$	$\vec{I} = -\sigma\vec{\nabla}\phi$
Conservation law	$\vec{\nabla}.\vec{q} = -S_h$	$\vec{\nabla}.\vec{I} = -S_e$

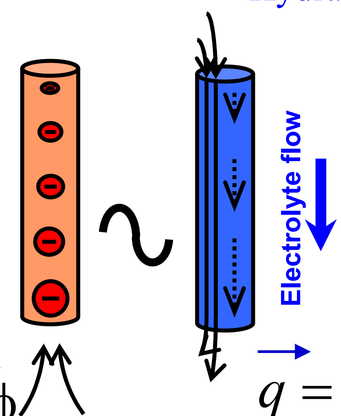
Electrokinetic coupling:

$$\begin{cases} \vec{q} = -K\vec{\nabla} h + \cancel{C\sigma\vec{\nabla}\phi} \\ \vec{I} = C\sigma\vec{\nabla} h - \sigma\vec{\nabla}\phi \end{cases}$$

where $$C = \left.\frac{dh}{d\phi}\right|_{I=0}$$

Electric Current

Hydraulic Current



$\vec{I} = -\sigma\vec{\nabla}\phi$

$\vec{q} = -K\vec{\nabla} h$

Principle from the macroscopic scale: thermodynamic dissipation is nearly linear

Thermodynamic approach:

	Hydraulic head gradient	Electric field
Forces	$\vec{X}_1 = -\vec{\nabla} h$	$\vec{X}_2 = -\vec{\nabla}\phi$

Total Flux of coupled forces

$$\vec{J}_i = f_i(\vec{X}_j) \approx \sum_j L_{ij}\vec{X}_j$$

$$\begin{cases} \vec{q} &= -K\vec{\nabla} h - L_{12}\vec{\nabla}\phi \\ \vec{I} &= -L_{21}\vec{\nabla} h + \sigma\vec{\nabla}\phi \end{cases}$$

Dissipation function (of the entropy at constant temperature):

$$D \approx \sum_{ij} L_{ij}\vec{X}_i.\vec{X}_j = \vec{q}.\vec{\nabla} h + \vec{I}.\vec{\nabla}\phi \Rightarrow L_{12} = L_{21} = -C\sigma$$

Finally:

$$\vec{\nabla}.(\sigma\vec{\nabla}\phi) = -\vec{\nabla}.(C\sigma\vec{\nabla} h)$$

Interpreting SP = Solving conservation laws

1. Conservation of hydraulic flux

$$\vec{\nabla}.(-K\vec{\nabla}h) = -S_h$$

2. Conservation of electric flux
with electrokinetic source term

$$\vec{\nabla}^2\phi = -S_e$$

Both C and σ constant ⇨ Poisson equation + simple connection to hydraulic

$$S_e = C\vec{\nabla}^2 h = \tfrac{C}{K} S_h - C\vec{\nabla}\ln K.\vec{\nabla}h$$

Solutions to the interpretation of SP based on « homogeneous » Green functions and deconvolution:

Inversion of Continuous wavelet transforms (Gibert & Pessel, Sailhac & Marquis 2001
Tomography of Charge Occurrence Probability (Patella 1997)
Top aquifer tomography (Fournier 1989, Birch 1993, Revil et al. 2003), etc.

$$\nabla^2 G = -\delta$$

$$\phi = G * S_e$$

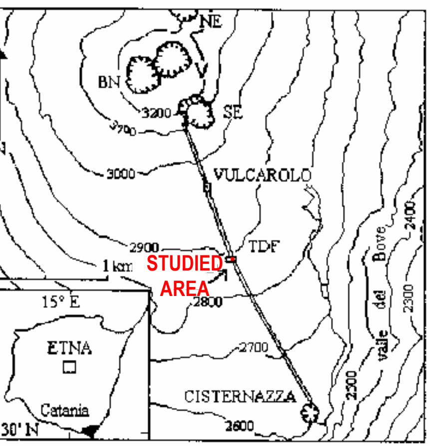
Wavelet transforms and hydrothermal application

One can use the Poisson wavelet basis, this means:

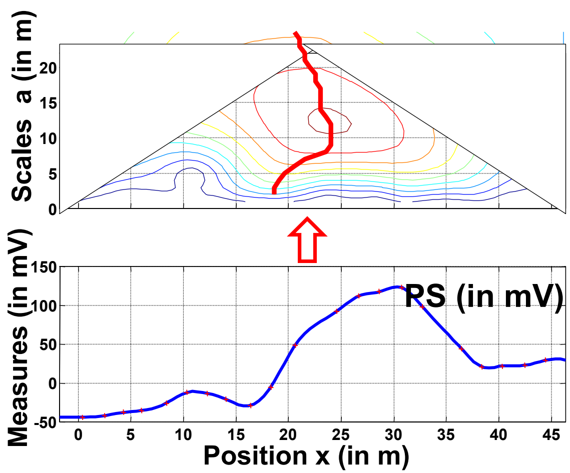
- Correlation coefficients with Green's function = wavelet transform coefficients
- analytic relations between fluid flow potentials and SP wavelet transform exist

pplication : Hydrothermal circulation at Etna volcano

Fissure Vulcarolo (Etna)

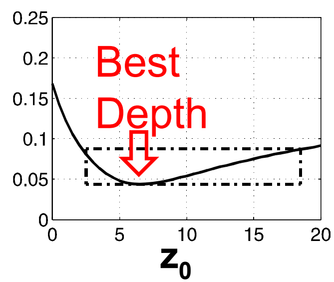


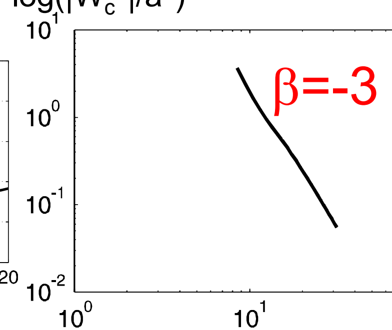
SP wavelet transform



Inversion by adjusting a power law (linear least-squares in log-log)

$$\left|W_c^2\right| \sim \frac{mC}{K} a^2 (a+z_0)^{\beta}$$

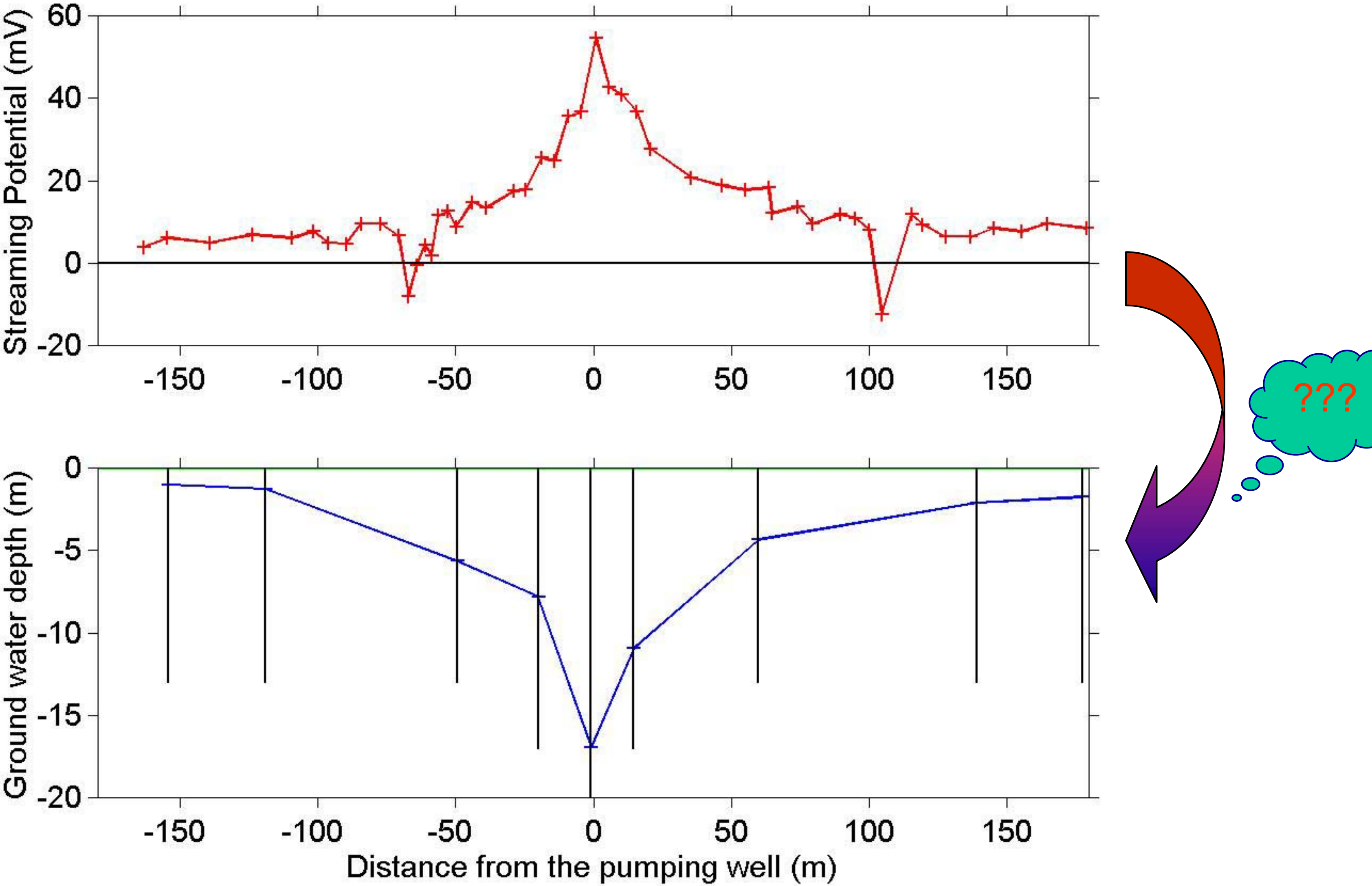




depth = 3 to 18 m
intensity = 2400 mV/m
flux ~ 0.1m²/h

(Fig. from Sailhac and Marquis, 2001)

Pumping test: Bogoslovski data case



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Pumping test: three methods

Inversion of Continuous wavelet transforms (Gibert & Pessel, Sailhac & Marquis 2001

W(x,a) = Convolution product with a Green function, then fit for scaling laws

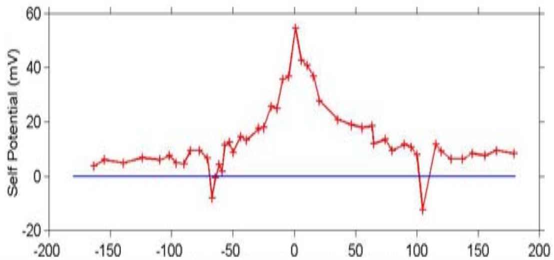
Tomography of Charge Occurrence Probability (Patella 1997)

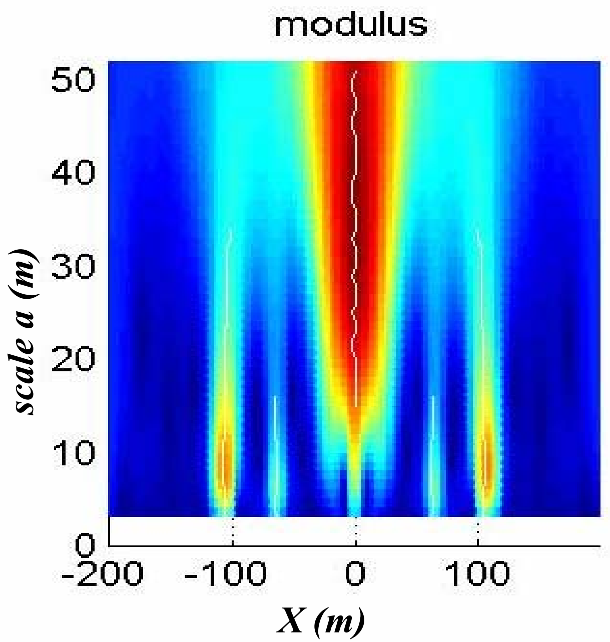
COP(x,z) = Cross correlation with a Green function, then display versus (x,z)

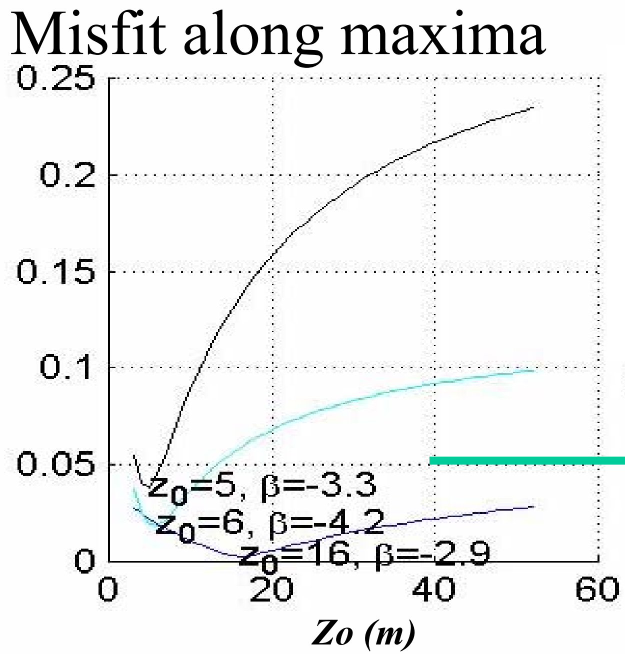
Top aquifer tomography (Fournier 1989, Birch 1993, Revil et al. 2003), etc.

$\alpha(x,z)=COP(x,z)/(h(x)-h_0)$ = Local normalization of the COP, then draw iso-α

Using wavelets





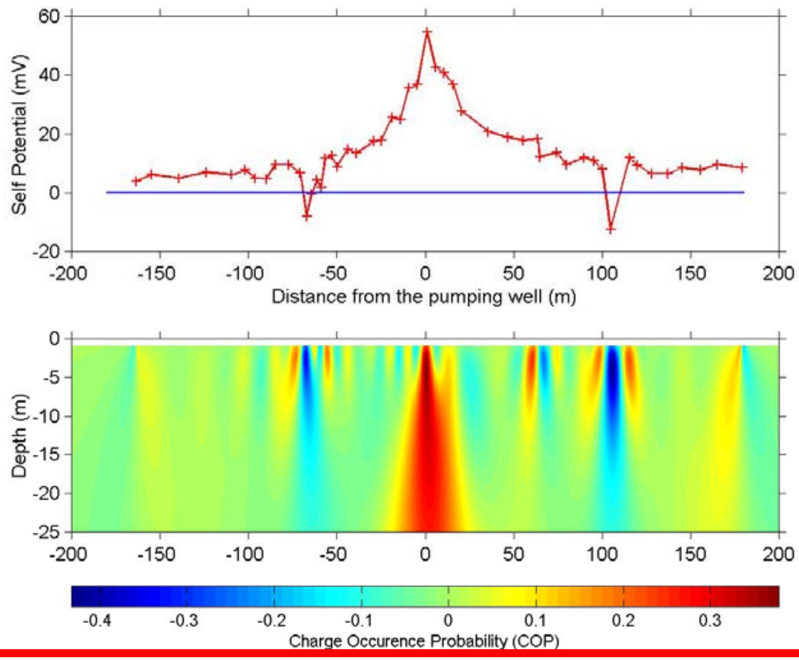


$$\left|W\right| \sim a^{\gamma}(a+z_0)^{\beta}$$

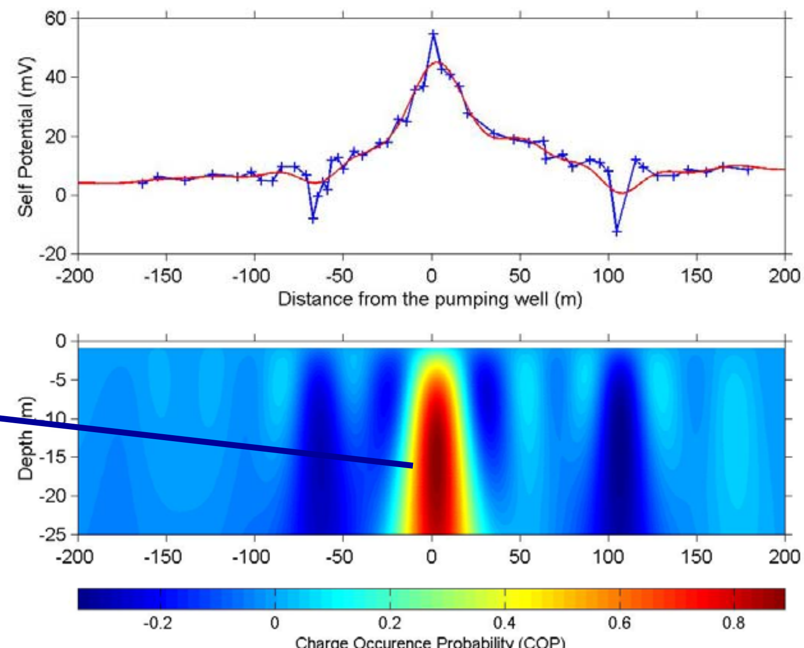
Z_0 = 16 ± 3 m

β = -3

Using correlation COP of Patella and low pass



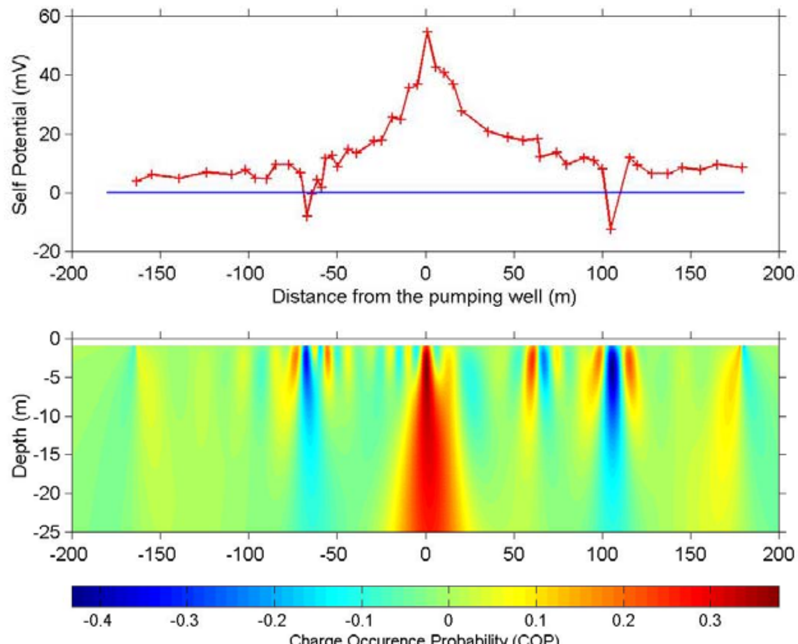
Low Pass at 10 m



Z_0 = 16 ± 3 m

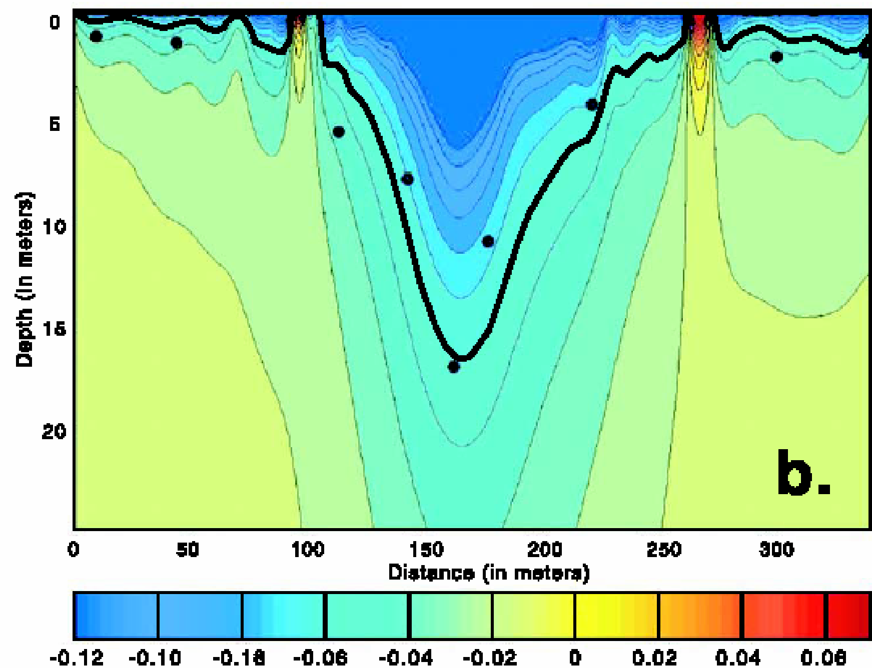
x_0 = 3 ± 5 m

Using correlation COP of Patella and normalize



Normalize (Revil et al. 2003)

$$\alpha = \frac{COP}{h - h_0}$$



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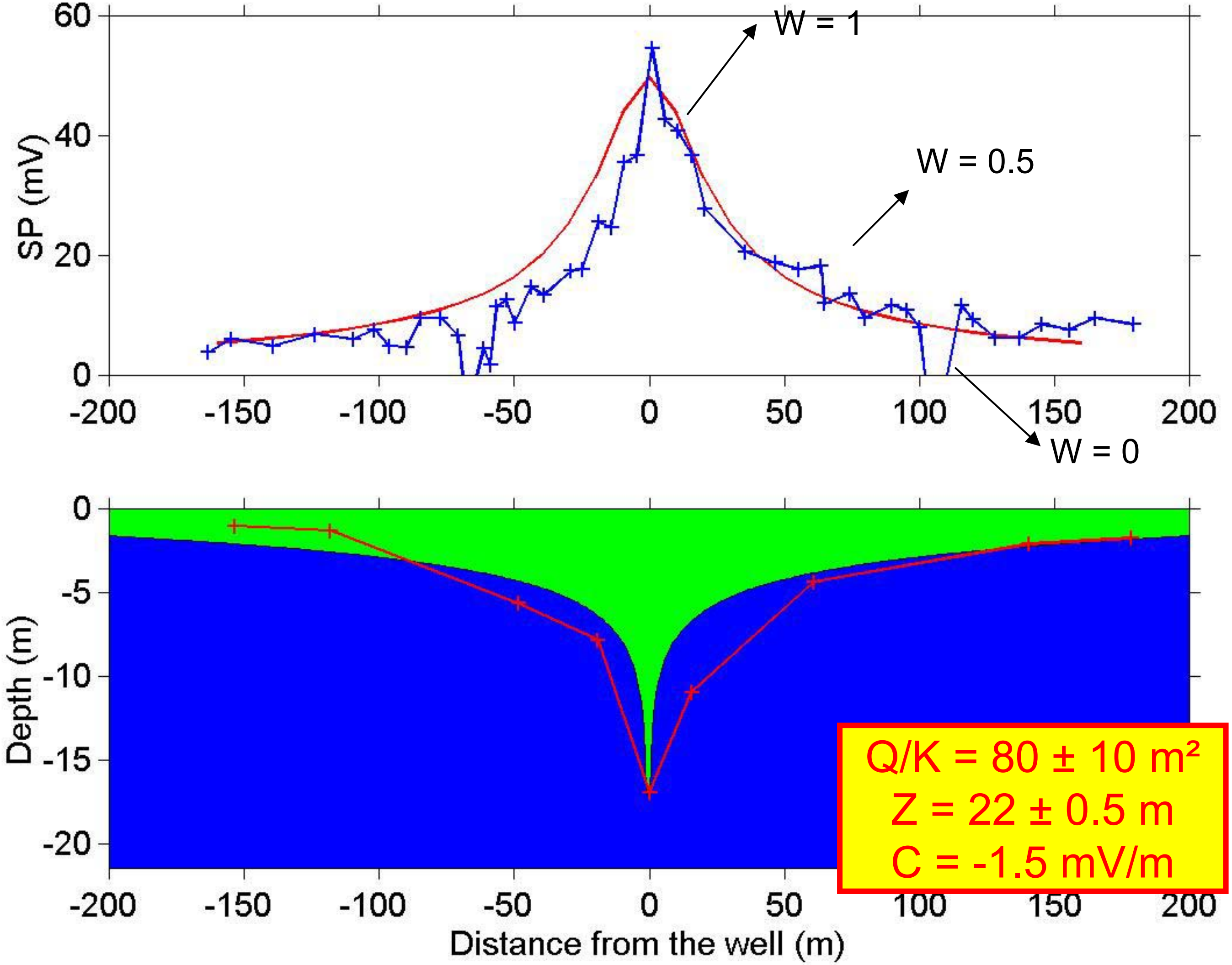
Genetic algorithm and pumping test application

- Use full inverse modelling including pumping and soil parameters C, Q/K, ..



(Data from Bogoslovsky and Ogilvy, 1973
Fig. from Darnet, Marquis, and Sailhac, 2003)

Unsaturated soil parameters: non-linearity!

Target Processes: Infiltration, Aquifer recharge, capillary fingering, etc.

Target parameters:

Corey-Brooks

$$K = K_s \left(\frac{\theta - \theta_r}{\theta_s - \theta_r} \right)^{3+2/\lambda_b}$$

Infiltration = Diffusion of water θ (Richard's Equation):

$$\frac{d\theta}{dt} = \nabla.(D\nabla\theta) - \frac{\partial K}{\partial z}$$

Van Genuchten

$$\psi = \psi_b \left[\left(\frac{\theta - \theta_r}{\theta_s - \theta_r} \right)^{-c/\lambda_v} - 1 \right]^{1/c}$$

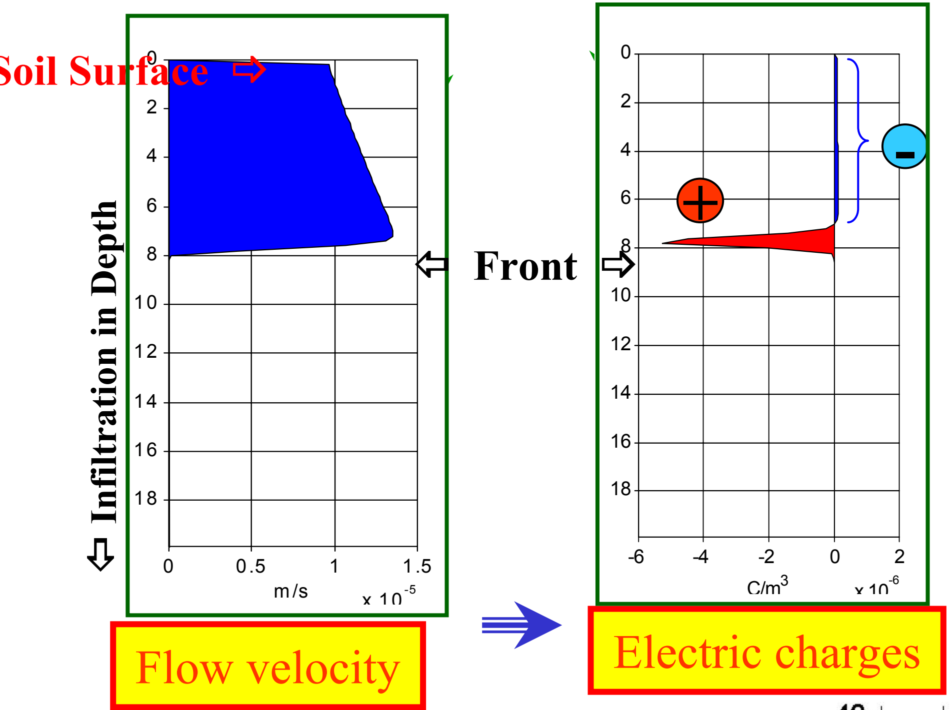
Gardner

$$K(\Psi) = K_S \, e^{\alpha\Psi}$$

Russo

$$S_e(x,z) = \left\{ e^{\frac{1}{2}\alpha\,\psi\, x,z)} \left[1 - \frac{1}{2}\alpha\,\psi(x,z) \right] \right\}^{2/2+m}$$

Unsaturated flows: 1D Transient infiltration



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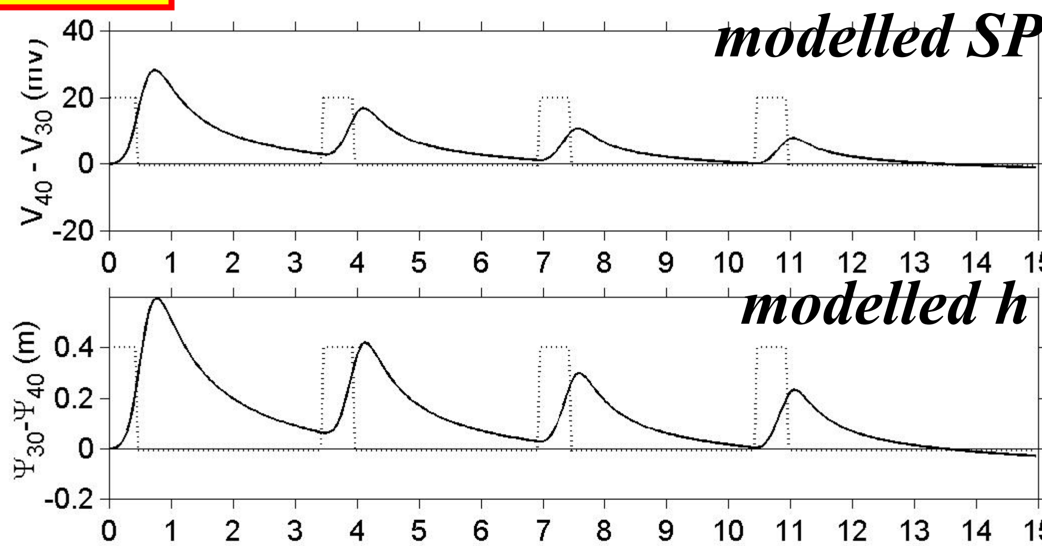
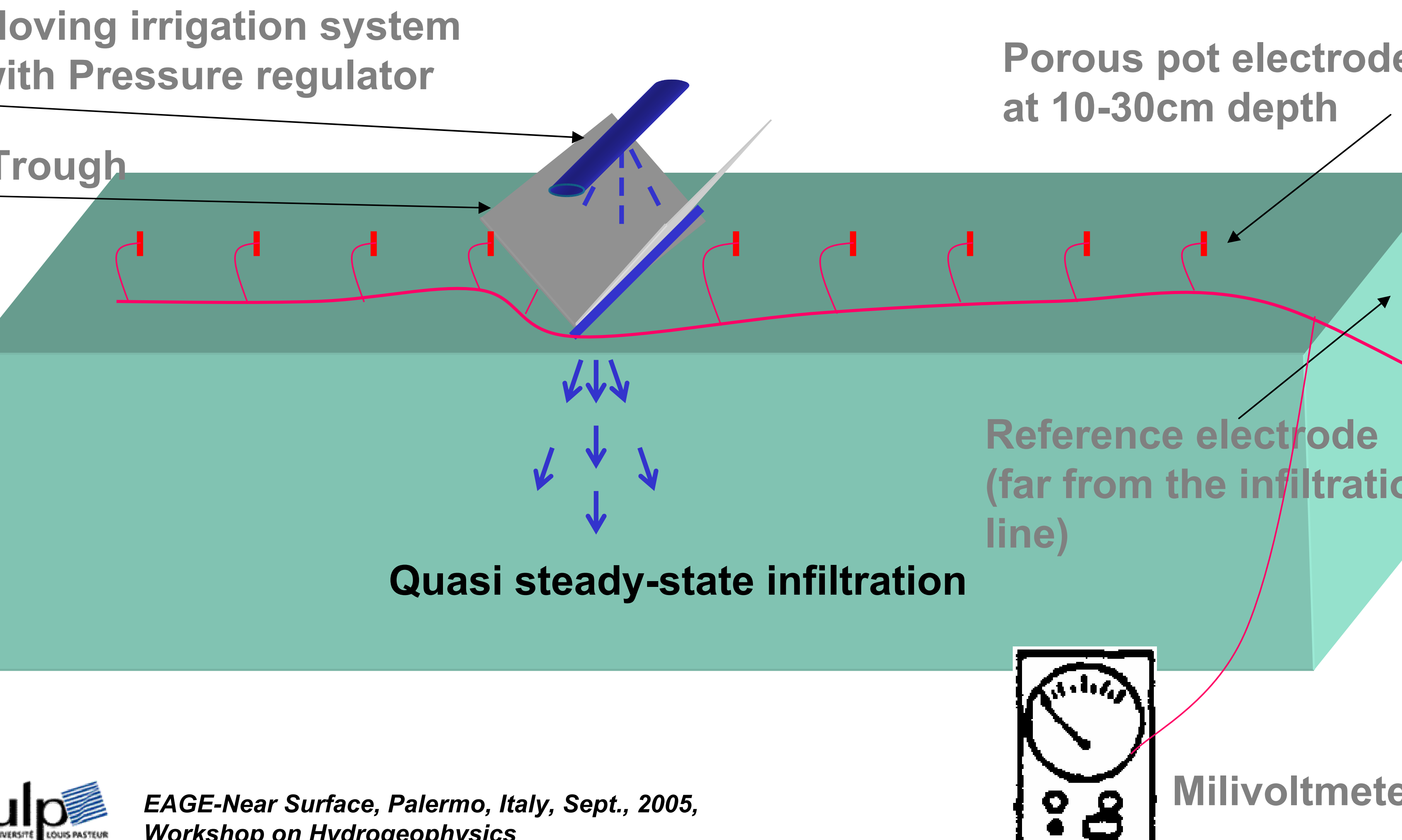
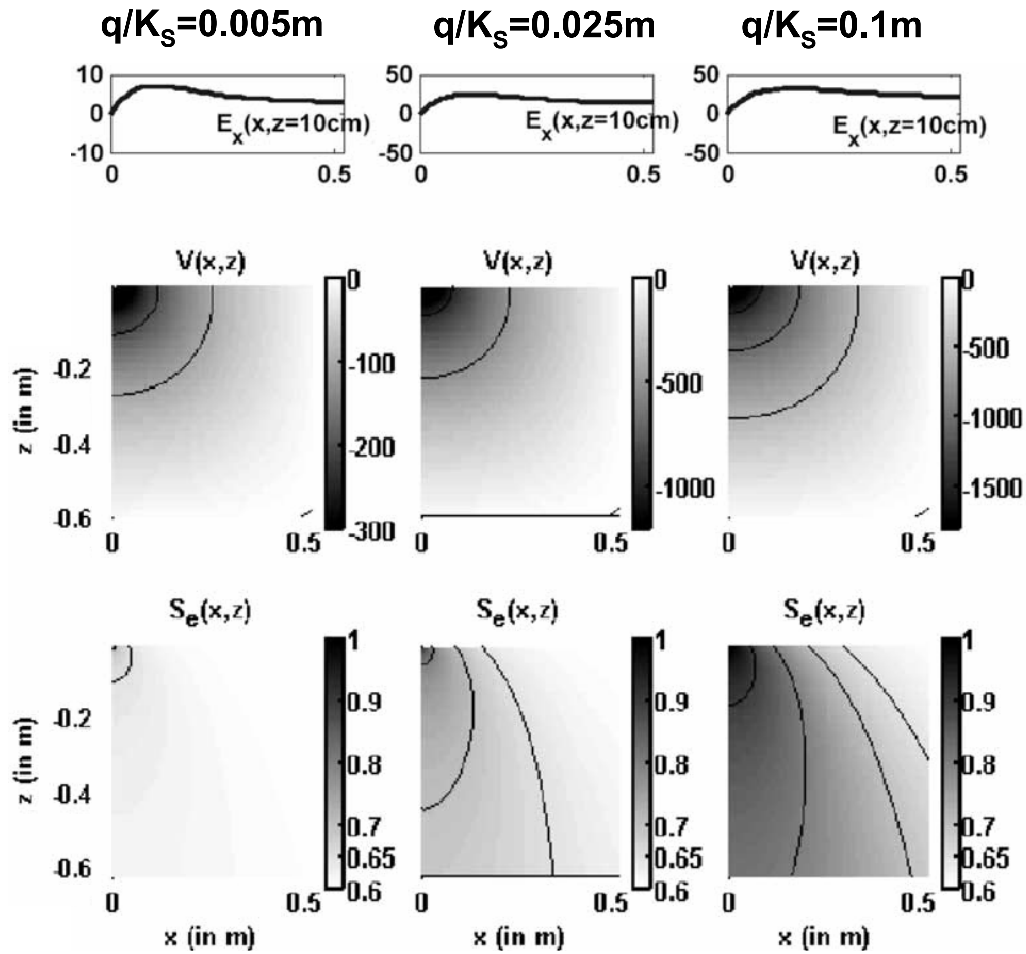


(Fig. from Darnet and Marquis, 2003)

Unsaturated flows: 2D infiltration



Unsaturated flows: 2D infiltration (simulations)



Cross sections of 3 source strength infiltration simulations q/KS ranges from 5mm to 10cm, show realistic effective water saturation Se and measurable electrokinetic potential V in depth (from 0 to 60 cm), and its horizontal derivative at 10 cm depth (horizontal electric field Ex).

Conclusions & Perspectives

SP interpretation can be applied for fluid flow characterization in to a variety of situations: fast interpretation techniques can already be used in the approximation of homogeneous media, otherwise inverse schemes (e.g. genetic) can be used.

Other ongoing works which will improve the method:

- **numerical modelling from the pore size up to the macroscopic scales (e.g. Titov et al. 2002)**

- **sample scale electrokinetic behavior in faults and unsaturated porous media (e.g. Lorne et al. 1999, Guichet et al. 2003)**

- **sand box experiments (e.g. Revil et al. 2002, Maineult et al. 2004)**

- **applications in media in which additional electric conductivity tomography shows heterogeneities (e.g. Béhaegel et al. 2005)**

Inversion

Inversion – Modèle direct 3D

Hypothèses: 1) Nappe libre homogène
2) Ecoulement radial cylindrique (hypothèse de Dupuit)
3) Régime permanent

Mass conservation:

$$\nabla^2 h^2 = \frac{Q}{K}$$

Solution for BC: 1) $h(r = r_0) = h_0$

2) $\Phi)_{r0} = Q$

$$h = \sqrt{h_0^{\,2} + \frac{Q}{\pi K}\ln\left(r/r_0\right)}$$

Electric current conservation:

$$\nabla^2 V = C\nabla^2 h$$

SP on the ground:

$$V(r, z=0) = \frac{CQ^2}{2\pi^3 K^2}\int_0^{2\pi}\int_0^{\infty}\int_{Z-h(r')}^{Z}\frac{\left(h_0^{\,2} + \frac{Q\ln(r'/r_0)}{\pi K}\right)^{-3/2}}{r'\left(\left(r\cos\theta - r'\right)^2 + \left(r\sin\theta\right)^2 + z'^2\right)^{1/2}}\,d\theta\, dr'\, dz'$$

Cas d'un milieu homogène : Equation de poisson et solutions par convolution avec des solutions à sources locales (fonctions de Green)

Equation de Poisson : $\nabla^2\phi = -S$ $\Leftrightarrow$ $\nabla^2 G = -\delta$, $\phi = G * S$

Sources : $S = C\nabla^2\varphi$

2D Homogène :

$$G(x,z) = -\ln(x^2+z^2)$$

$$\begin{cases} P_z(x) = \partial_z G(x,z) = \dfrac{-2z}{x^2+z^2} \\ I_x(x,z) = \partial_x G(x,z) = \dfrac{-2x}{x^2+z^2} \end{cases}$$

3D Homogène :

$$G(x,y,z) = \frac{1}{(x^2+y^2+z^2)^{1/2}}$$

$$\begin{cases} P_z(x) = \dfrac{z}{(x^2+y^2+z^2)^{3/2}} \\ \ldots \end{cases}$$

Transformée en ondelettes complexes (de ϕ) : en et fonctions complexes de courant de fluide

Potentiel hydraulique : $\phi(x,y)$ => fonction de courant $\psi=H[\phi]$

Potentiel complexe d'écoulement : $f(\zeta)=\phi+i\,\psi$ de variable complexe $\zeta = x + i$

Transformée en ondelette des PS : $W_{c\,[\phi]}^{\gamma}(x,a) = C a^{\gamma} f^{(\gamma)}(\zeta)$

Type d'écoulement	*Fonction de courant*	*Coefficient d'ondelettes des données*
Flot uniforme de direction α	$f(\zeta) = -\, q_0\, e^{-i\alpha}$	$W_c^{\gamma\geq 1}(x,a) = 0$
Source ou puits à ζ_0	$f(\zeta) = m\, \ln(\zeta-\zeta_0)$	$W_c^{2}(x,a) = -a^2\, Cm/K\, (\zeta-\zeta_0)^{-2}$
Rampe avec coin d'angle π/α **à** ζ_0	$f(\zeta) = q_0\, (\zeta-\zeta_0)^{\alpha}$	$W_c^{2}(x,a) = a^2\, Cq_0/K\, \alpha(\alpha-1)(\zeta-\zeta_0)^{\alpha}$

Etc… (calcul par sommation ou par transformation conforme)