fiva
International Research School of Water Resources

International PhD Course in
HYDROGEOPHYSICS

Hydrological – geophysical relationships

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Overview

In the course we will concentrate on electrical, electromagnetic and radar methods for hydrogeophysical investigations.

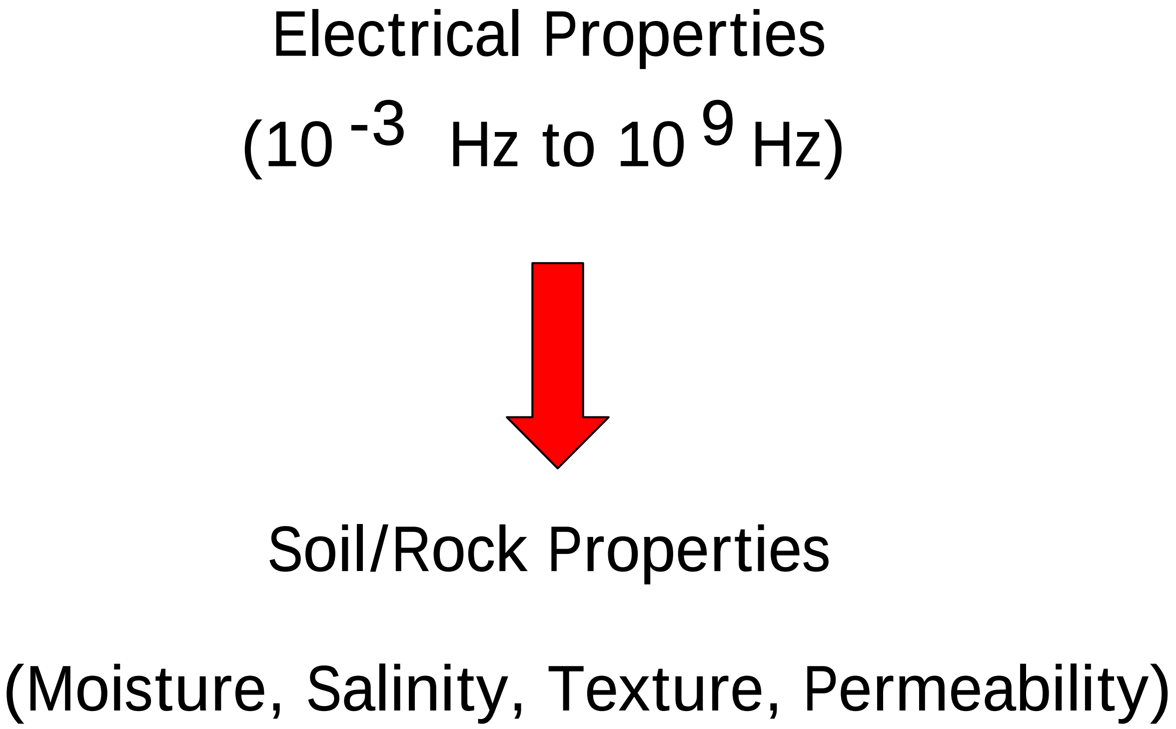
Here we discuss the known and assumed links between hydrological and electrical properties of the subsurface.

We cover:

- the basic definition of electrical properties;
- theoretical and empirical relationships;
- example applications.

Acknowledgement: many slides were provided by David Lesmes

Objective



Electrical Properties

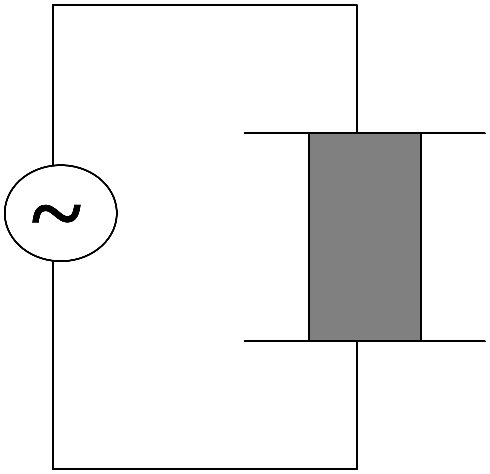
(10^{-3} Hz to 10^{9} Hz)

Soil/Rock Properties

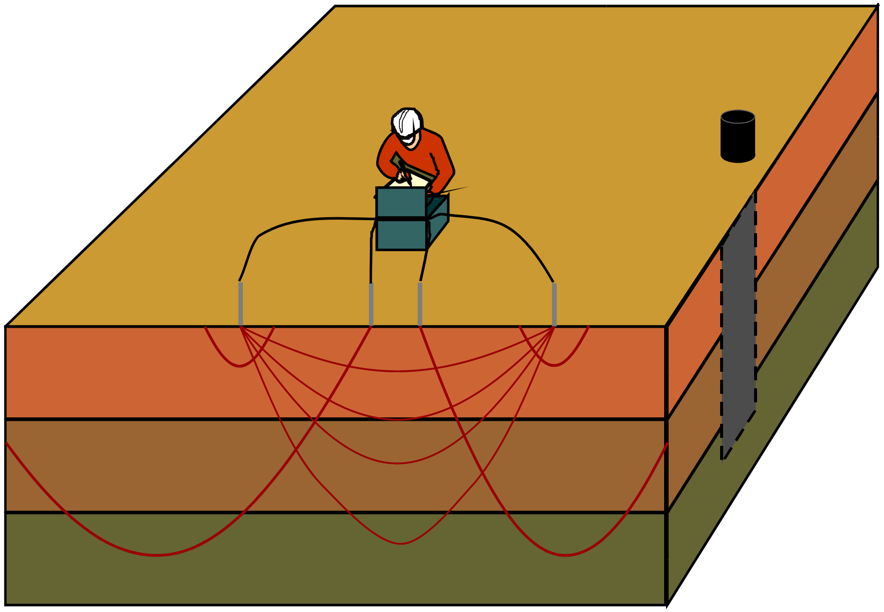
(Moisture, Salinity, Texture, Permeability)

Approach

Laboratory



Field



Hydrological properties

Porosity – ratio of pore volume (V_p) to total volume (V_t)

$$\phi = \frac{V_p}{V_t}$$

Moisture content – ratio of pore water volume (V_w) to total volume (V_t)

$$\theta = \frac{V_w}{V_t}$$

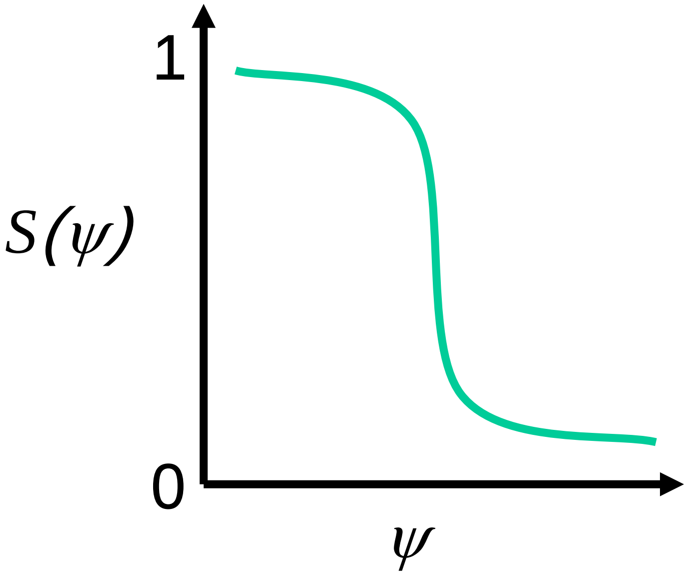
Effective saturation – ratio of ‘changeable’ moisture content to total ‘changeable’ moisture content.

$$S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r}$$

Hydrological properties

Effective saturation – a function of the pressure head of the pore fluid, for example in the van Genuchten (1980) model:

$$S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r} = \frac{1}{\left[1 + \alpha(\psi)^n\right]^m}$$



Hydrological properties

Hydraulic conductivity – controls the rate at which water moves through the porous media – is a function of the permeability (k_s), density (ρ_w) and viscosity (μ), :

$$K_s = \frac{k_s \rho_w g}{\mu}$$

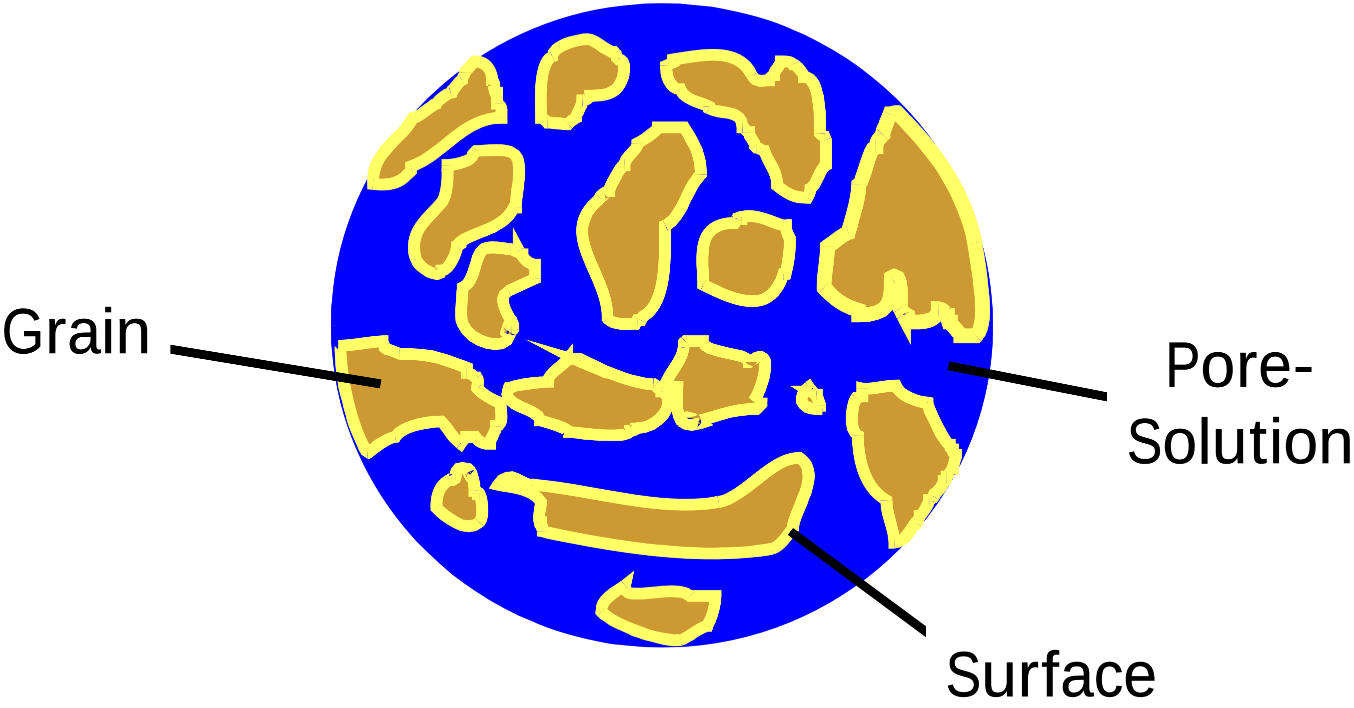
Many empirical models exist which relate the permeability to pore size or grain size or surface area.

For example the Kozeny-Carman equation

$$k_s = \frac{\phi^3}{(1-\phi)^2} \times \frac{1}{5S_{por}}$$

S_{por} = surface area per unit volume of solid

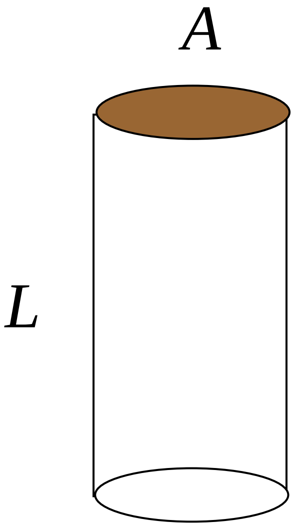
Electrical properties

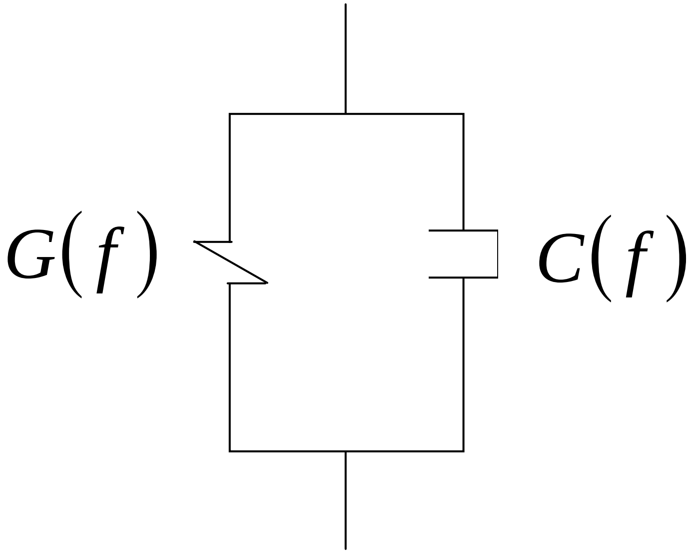


Electrical Transport = Flow + Storage

Electrical properties

- 0.001 Hz – 1kHz: Four-Electrode
- 100 Hz – 100 MHz: Two-Electrode
- 10 MHz – 1GHz: Transmission Lines





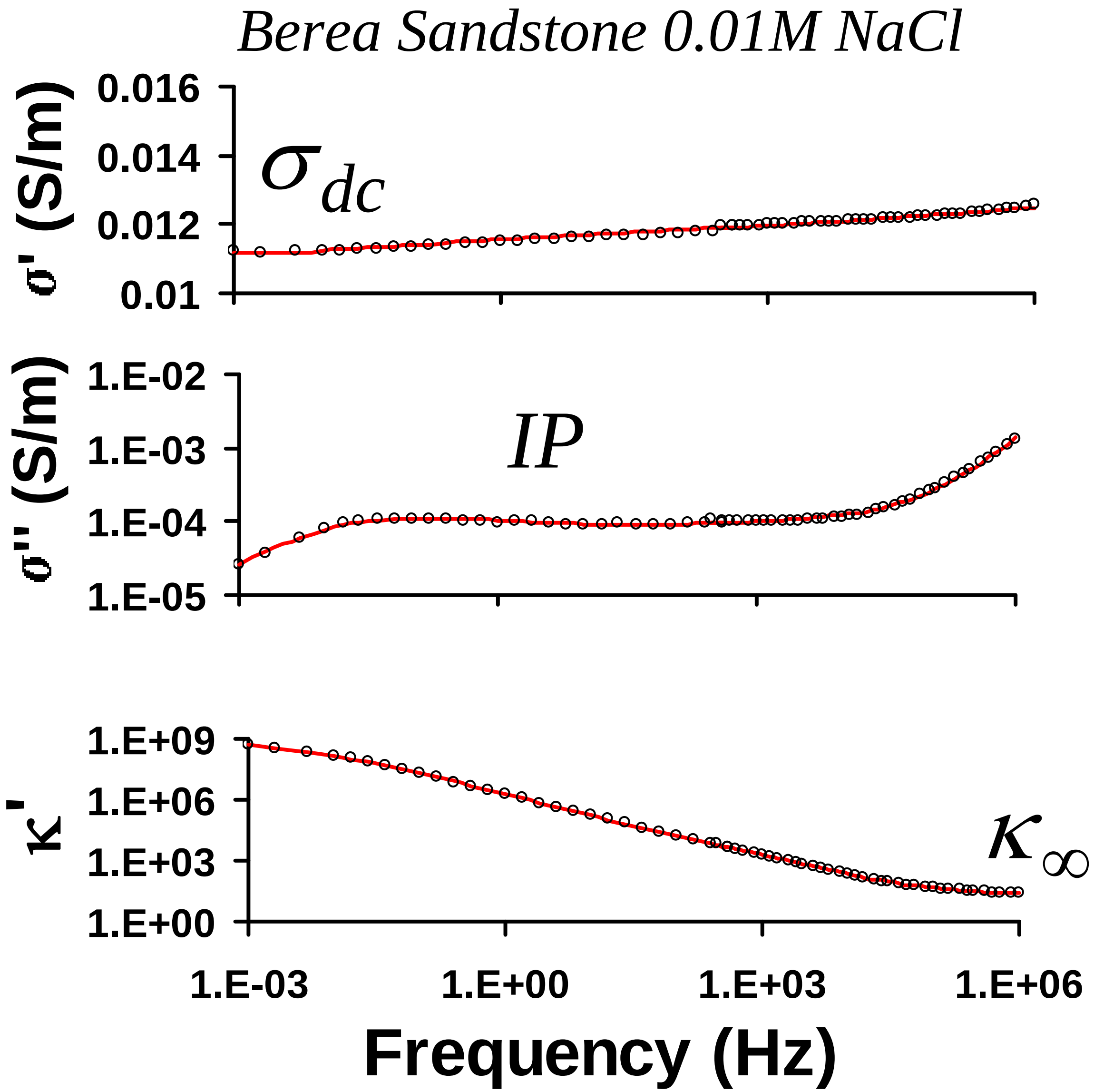
Flow - *conductivity*

$$\sigma(f) = G(f)\frac{L}{A}$$

Storage – *permittivity*

$$\varepsilon(f) = C(f)\frac{L}{A}$$

Conductivity and permittivity are complex variables



Flow

σ'

Storage

$$\kappa' = \sigma''/\omega\varepsilon_0$$

After Lesmes and Friedman (2005)

Electrical Parameters	Soil/Rock Properties
Dielectric (radar)	Water Content
Conductivity (resistivity, ground conductivity)	Salinity, Texture/ Lithology
Induced polarisation (IP)	Texture/Lithology, Surface Chemistry
Spectral induced polarisation (SIP) $\sigma^*(f)$	Grain/Pore Size?

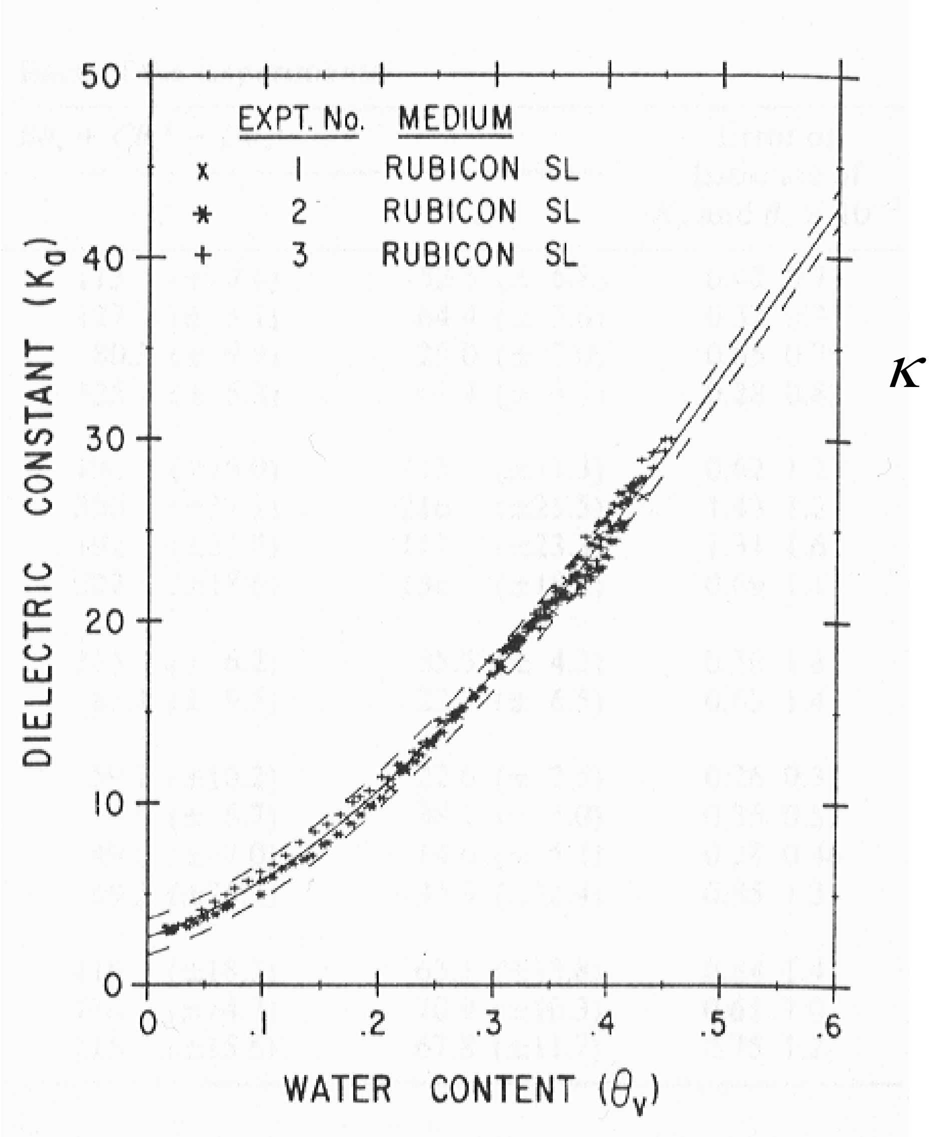
Relationships

Many petrophysical models exist which describe the relationships between geophysical and hydrological properties.

Some are semi-empirical and based on geometrical averaging some are purely empirical.

Here we concentrate on common models.

Permittivity – moisture content



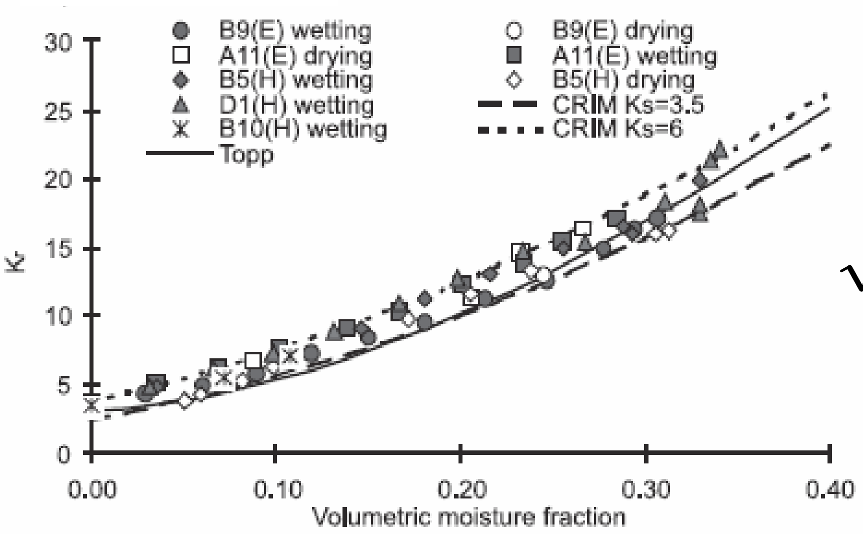
Topp et al. (1980)

$$\kappa = 3.03 + 9.3\theta + 146\theta^2 - 76.3\theta^3$$

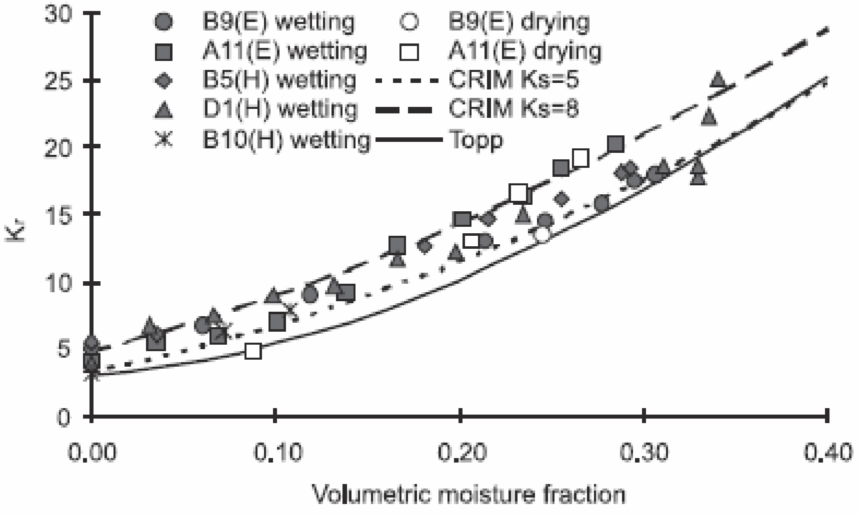
Widely used for time domain reflectometry (TDR) and some radar

Permittivity – moisture content

500 MHz



150 MHz



Complex refractive index model (CRIM)

$$\sqrt{\kappa} = \theta\sqrt{\kappa_w} + (\phi-\theta)\sqrt{\kappa_a} + (1-\phi)\sqrt{\kappa_s}$$

Mixing model based on individual components

$$\kappa_w = 81, \kappa_a = 1$$

$$5 \le \kappa_s \le 20$$

(typically)

After West et al. (2003)

Permittivity – moisture content

CRIM:

$$\sqrt{\kappa} = \theta\sqrt{\kappa_w} + (\phi - \theta)\sqrt{\kappa_a} + (1-\phi)\sqrt{\kappa_s}$$

General mixing model:

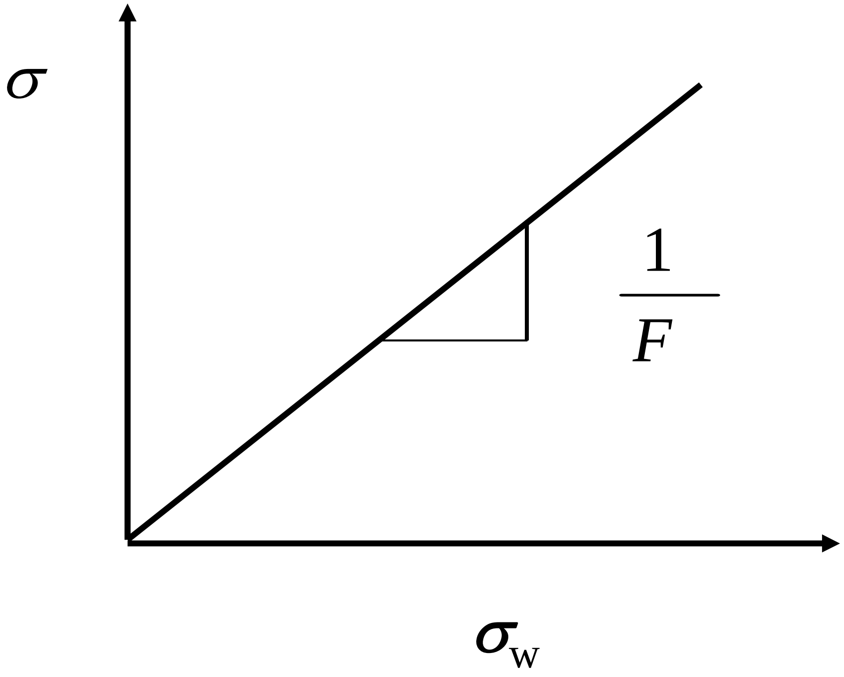
$$\kappa^a = \sum \Theta_i \kappa_i^a$$

κ_i	Dielectric constant on fraction i
Θ_i	Volume of fraction i
a	Limits are: 1 (perpendicular flow), -1 (parallel flow)

Resistivity/conductivity

Archie’s empirical law (Archie, 1942) is the most widely used.

$$\sigma = \sigma_w \phi^m S_w^n$$



Formation factor:

$$F = \frac{\sigma_w}{\sigma} = \phi^{-m}$$

Cementation index:

$$1.5 \leq m \leq 3$$

(typically)

Saturation index:

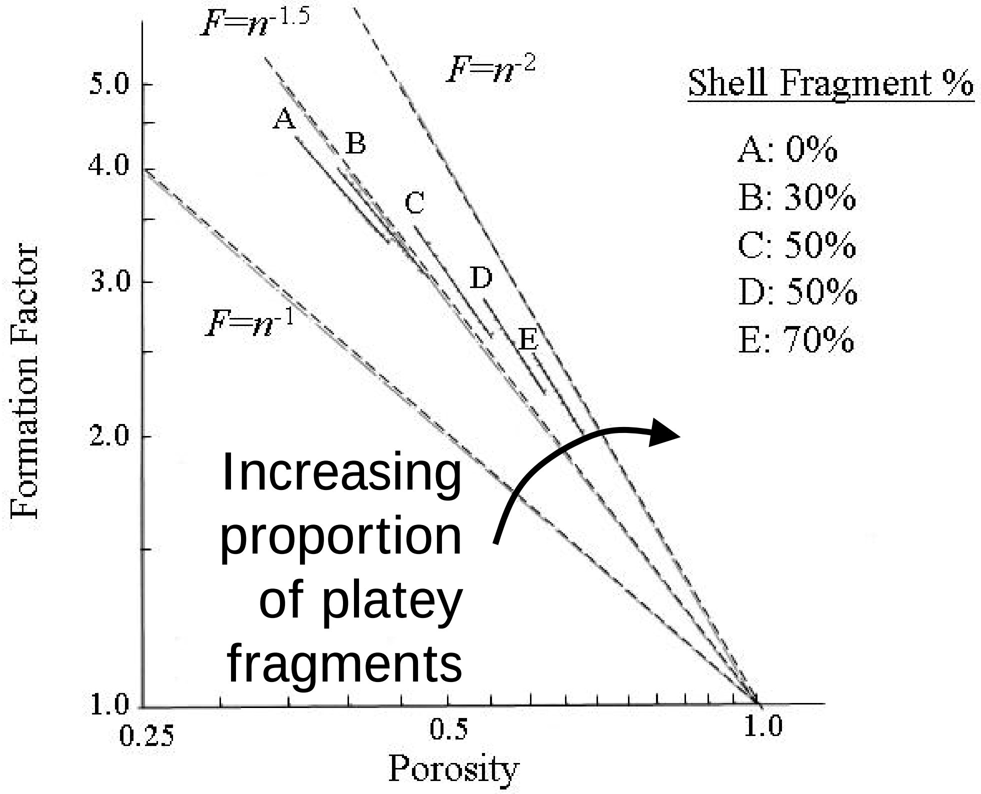
$$1.3 \leq n \leq 2$$

(typically)

Resistivity/conductivity

The cementation index increases as the grains become less spherical (Jackson, 1978)

$$\sigma = \sigma_w \phi^m S_w^n$$



Increasing proportion of platey fragments

Formation factor:

$$F = \frac{\sigma_w}{\sigma} = \phi^{-m}$$

Cementation index:

$$1.5 \le m \le 3$$

(typically)

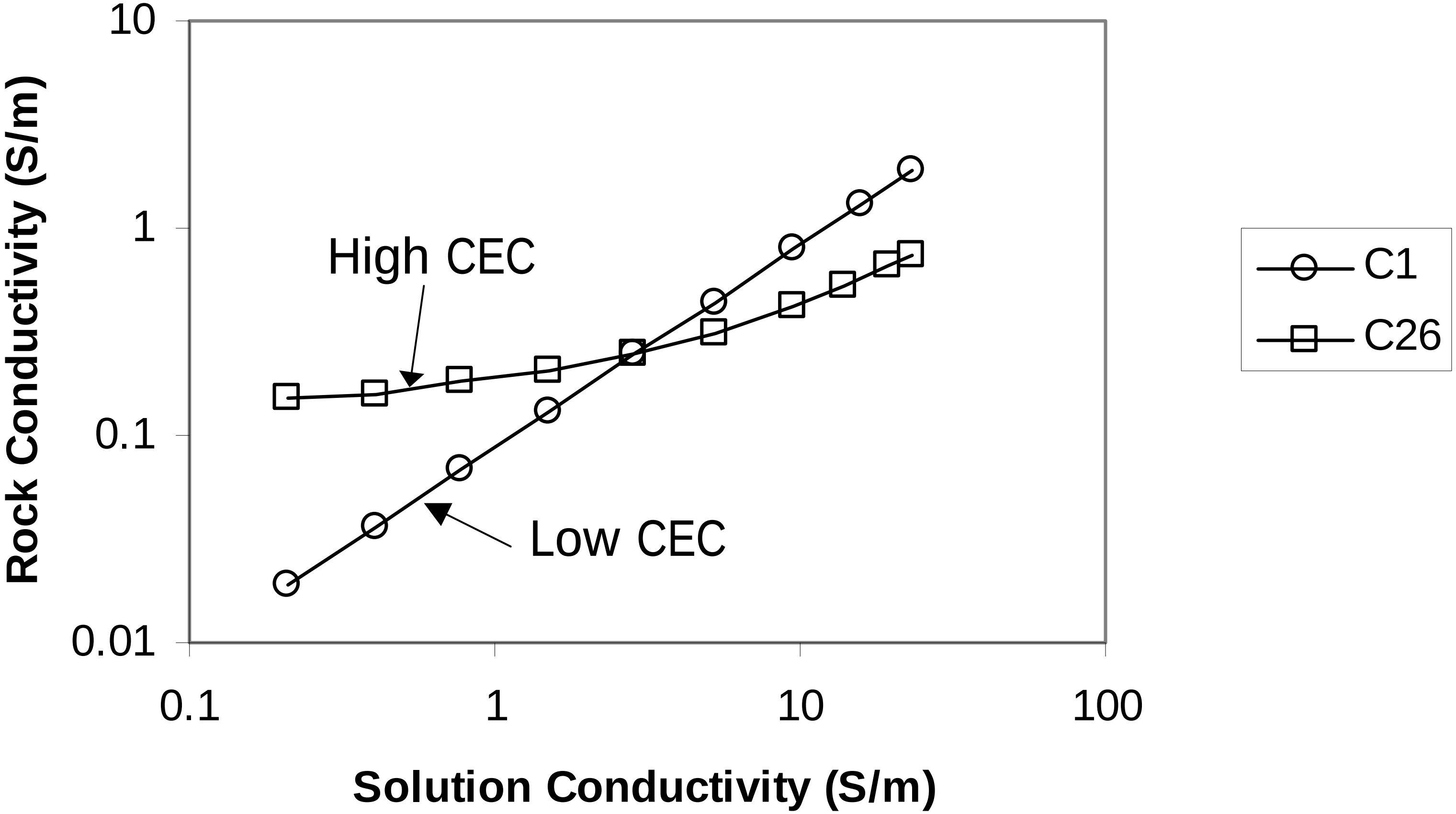
Saturation index:

$$1.3 \le n \le 2$$

(typically)

Resistivity/conductivity

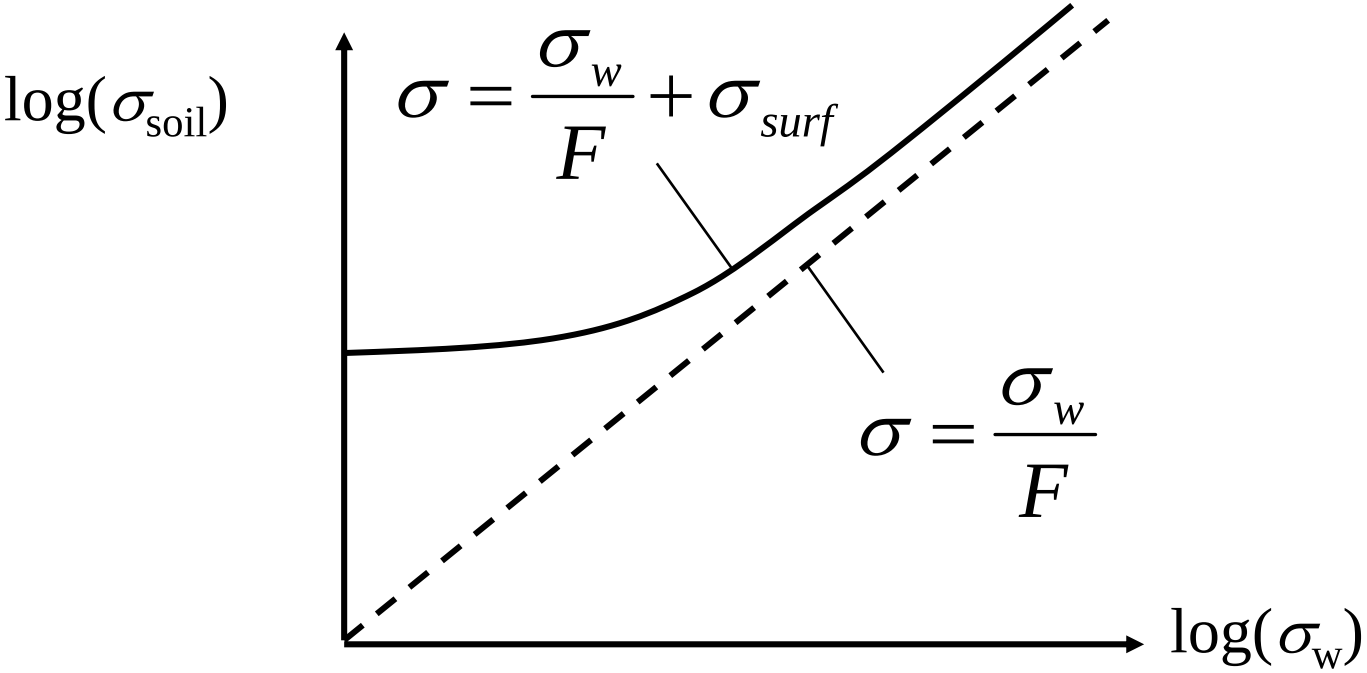
Archie’s law assumes no surface conductivity



After Lesmes and Friedman (2005)

Resistivity/conductivity

Archie’s law assumes no surface conductivity



Surface conductivity $\sigma_{surf} = \frac{BQ_v}{F}$ (Waxman and Smits, 1968)

B Equivalent ionic conductance of the clay exchange cations

Q_v Effective clay content

Resistivity/conductivity

In unsaturated porous media with surface conductivity:

$$\sigma = \sigma_w \phi^m S_w^n$$

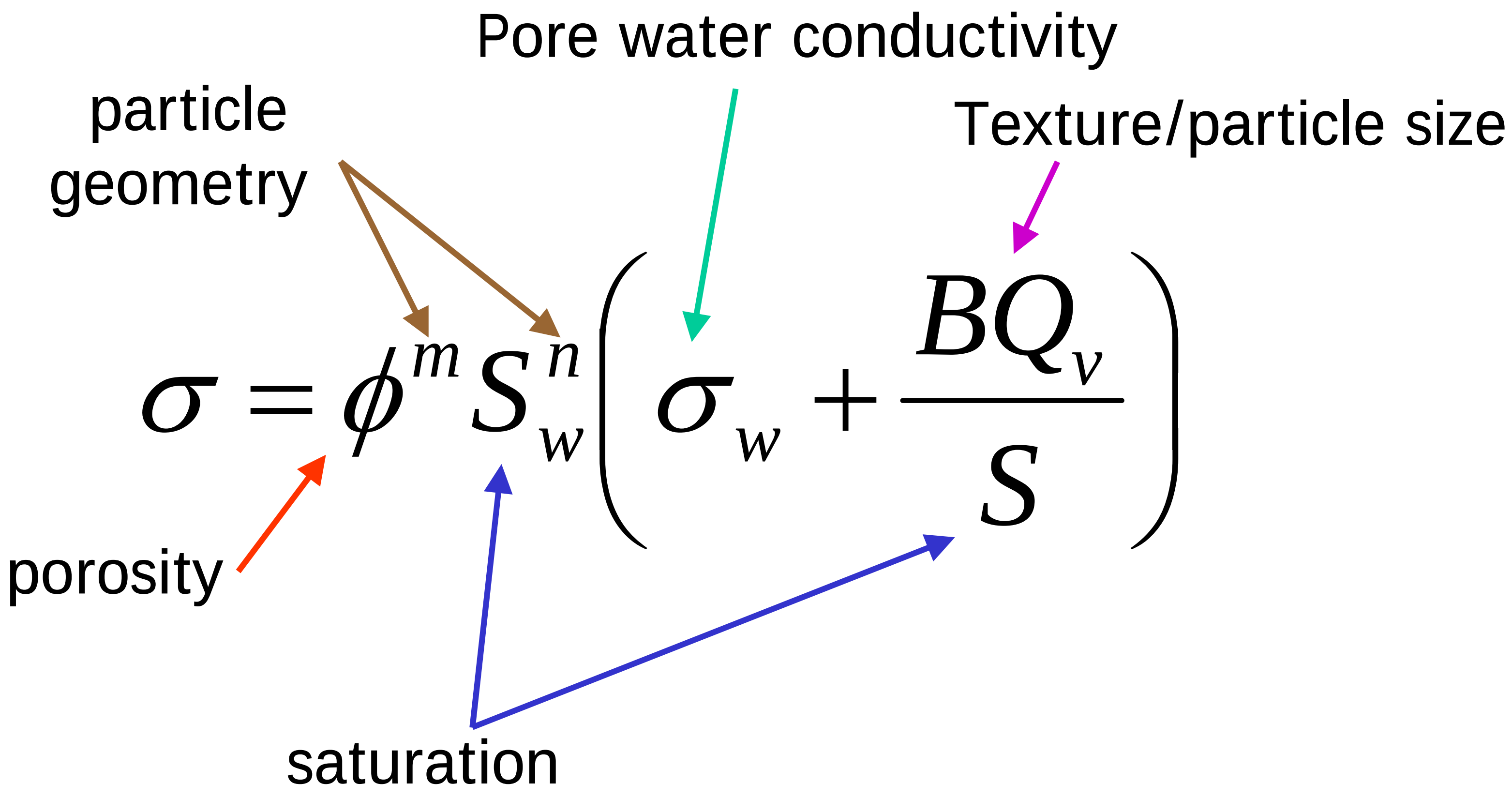
⇓

$$\sigma = \phi^m S_w^n \left(\sigma_w + \frac{BQ_v}{S} \right)$$

(Waxman and Smits, 1968)

Resistivity/conductivity

Conductivity is then a function of many hydrological properties:



Pore water conductivity

particle geometry

Texture/particle size

$$\sigma = \phi^m S_w^n \left(\sigma_w + \frac{BQ_v}{S} \right)$$

porosity

saturation

We need some way of separating out some of these effects

Induced polarisation (IP)

Advantages

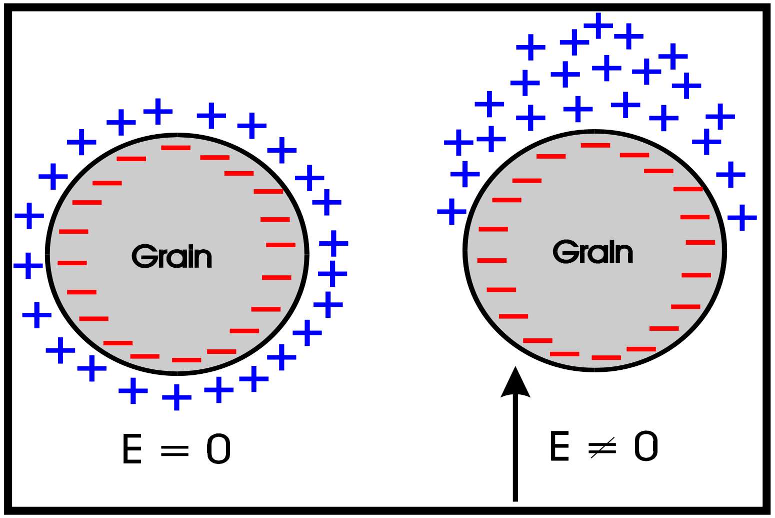
- A direct measure of surface properties of soils/rocks
- Permeability prediction?
- Contaminant detection/monitoring?

Disadvantages

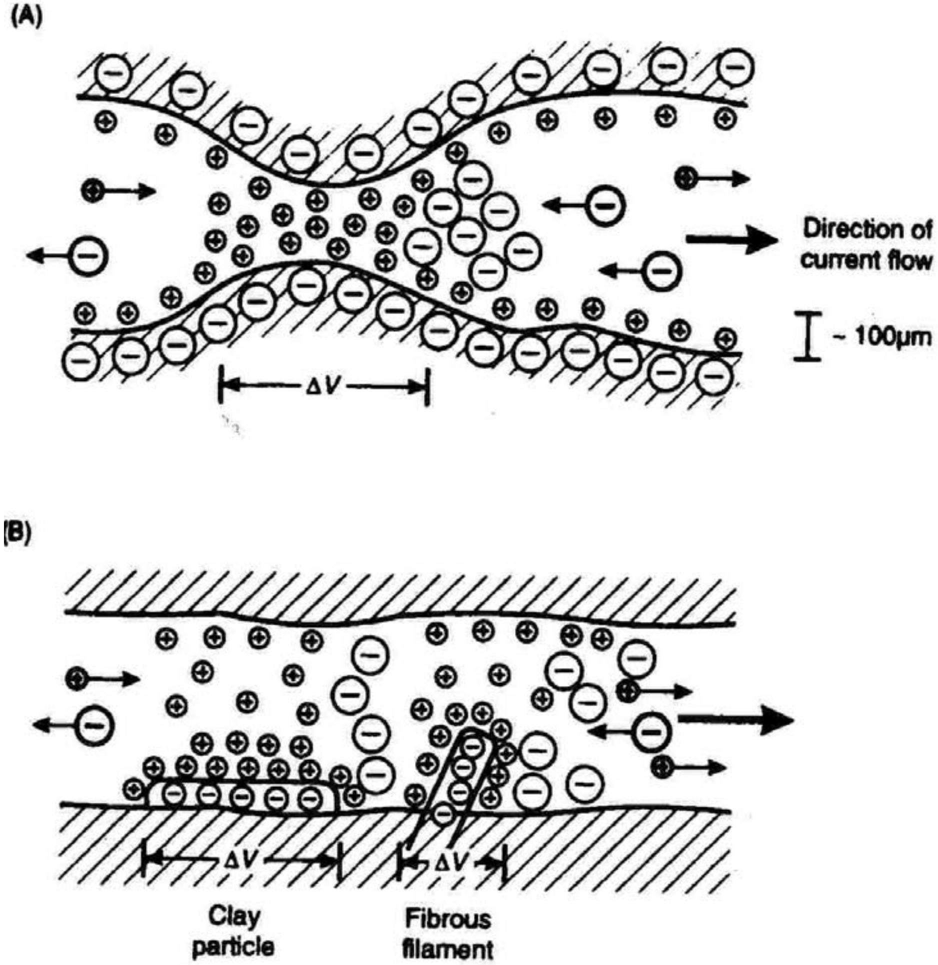
- Direct contact and low frequency method - slow
- Electrode and electromagnetic coupling errors
- Polarisation mechanisms not fully understood

Induced polarisation - polarisation mechanisms

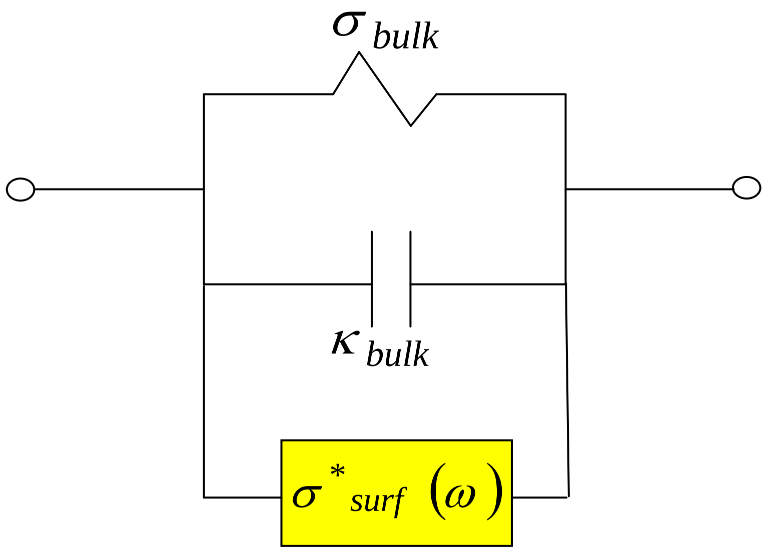
Electric-Double Layer (EDL) Polarisation (Schwarz, 1962)

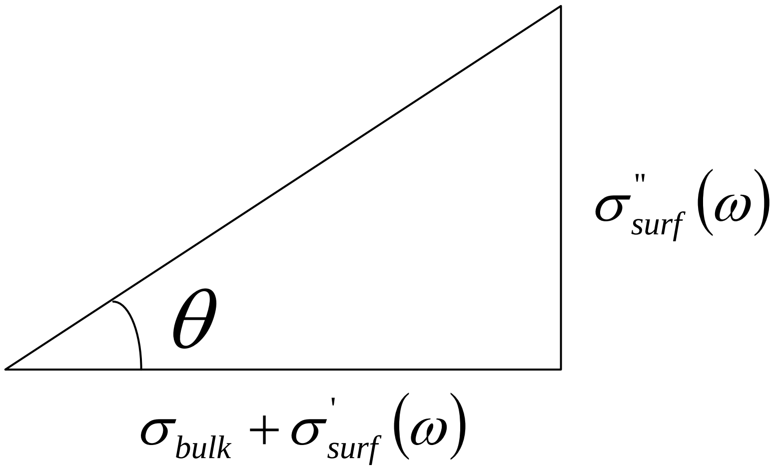


Membrane Polarisation
(Marshall & Madden, 1959)



Induced polarisation – equivalent circuit





$$\theta_{lowfreq} \cong \frac{\sigma''_{surf}(\omega)}{\sigma_{bulk}+\sigma'_{surf}(\omega)}$$

e.g., Vinegar and Waxman(1984)

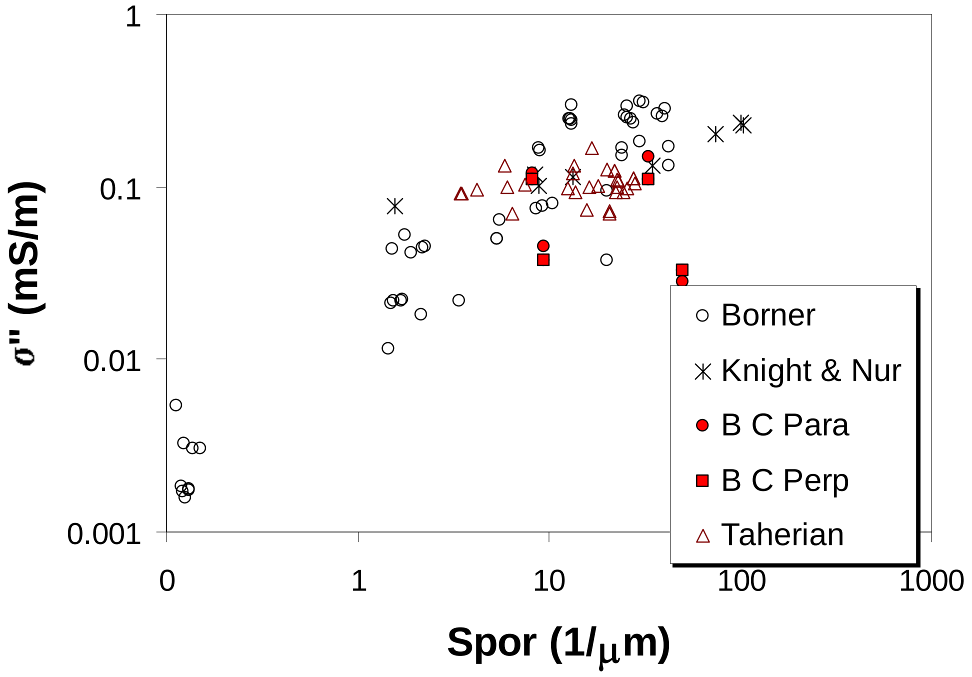
Induced polarisation – field parameters

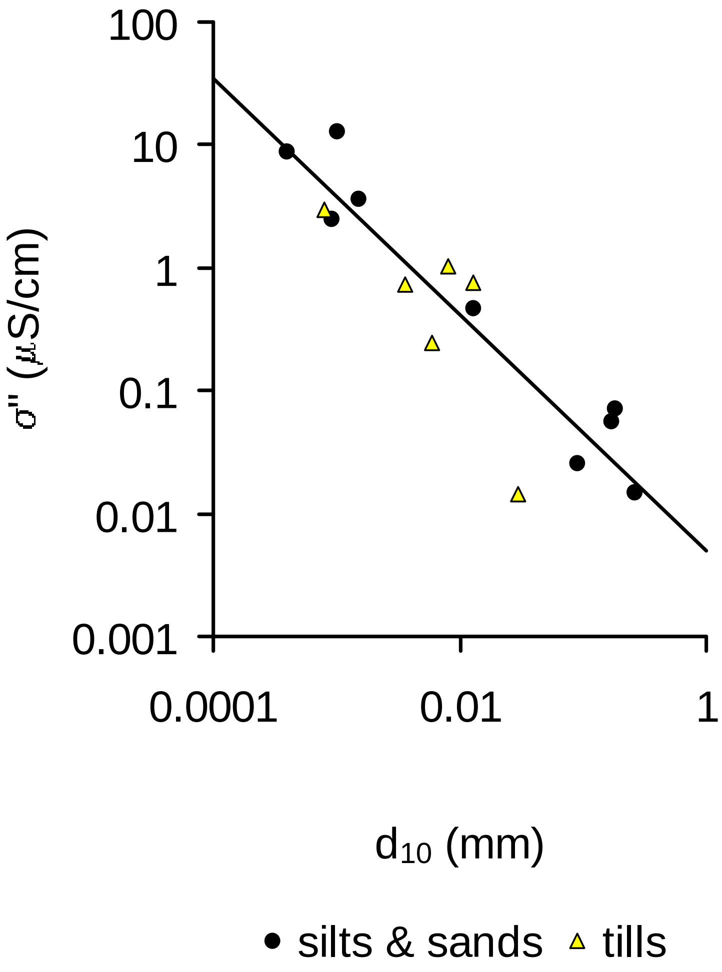
Phase angle $$\theta = \tan^{-1}\left(\sigma'' / \sigma'\right) \cong \sigma'' / \sigma.'$$

Perfect frequency effect $$PFE = 100 * \frac{\sigma(\omega_1) - \sigma(\omega_0)}{\sigma(\omega_0)}$$

Chargeability $$m = \frac{1}{V_{\max}(t_1 - t_0)} \int_{t_0}^{t_1} V(t)\,dt$$

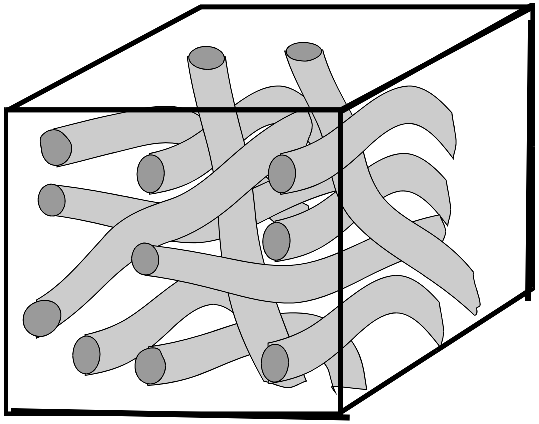
Induced polarisation – lithology effects





Slater and Lesmes (2001)

Permeability prediction



Tube/Crack Models

Kozeny-Carman

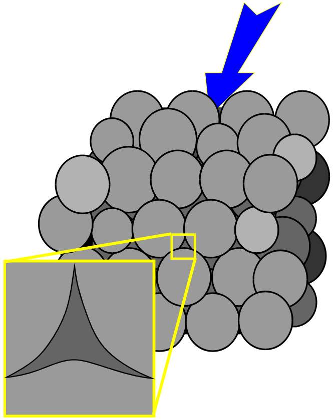
$$k = \frac{R_h^2}{aF} \cong \frac{1}{aFS_{por}^2}$$

Grain Models

Hazen (1911)

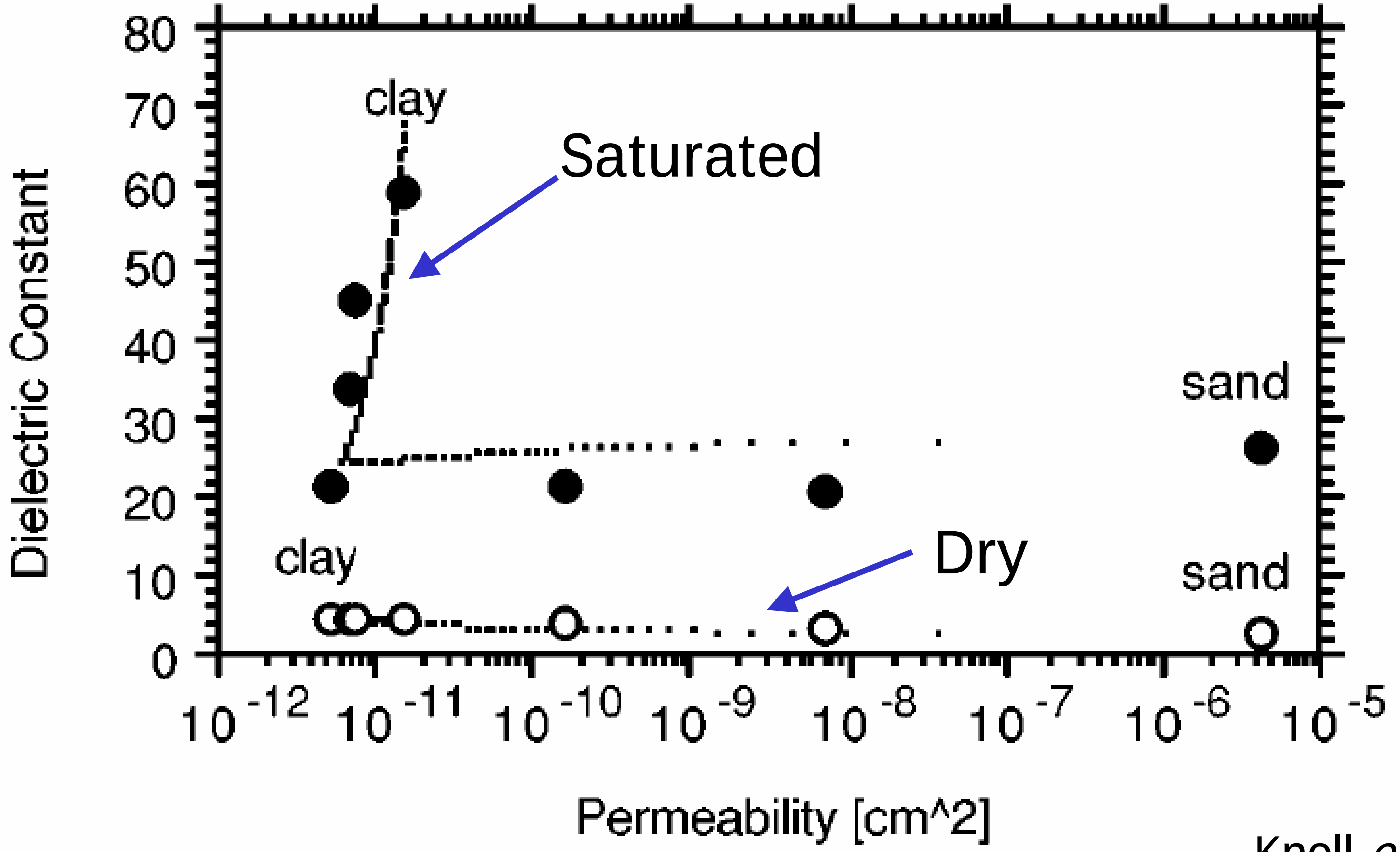
Krumbien and Munk (1943)

Berg (1970)



Permeability prediction – from permittivity

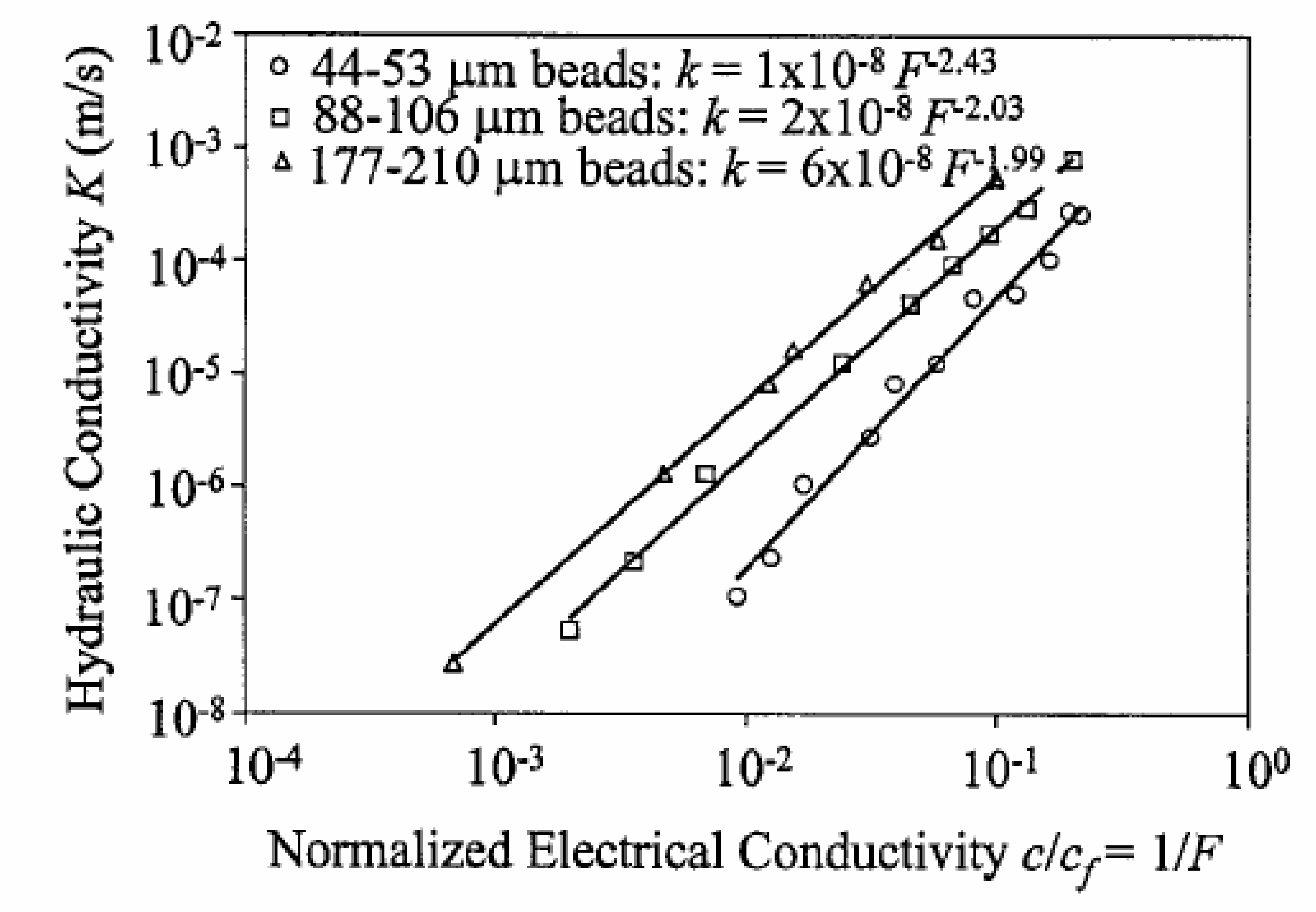
Since κ is a function of porosity (or moisture content) we may expect to see a correlation with permeability



Knoll *et al.* (1995)

Permeability prediction – from conductivity/resistivity

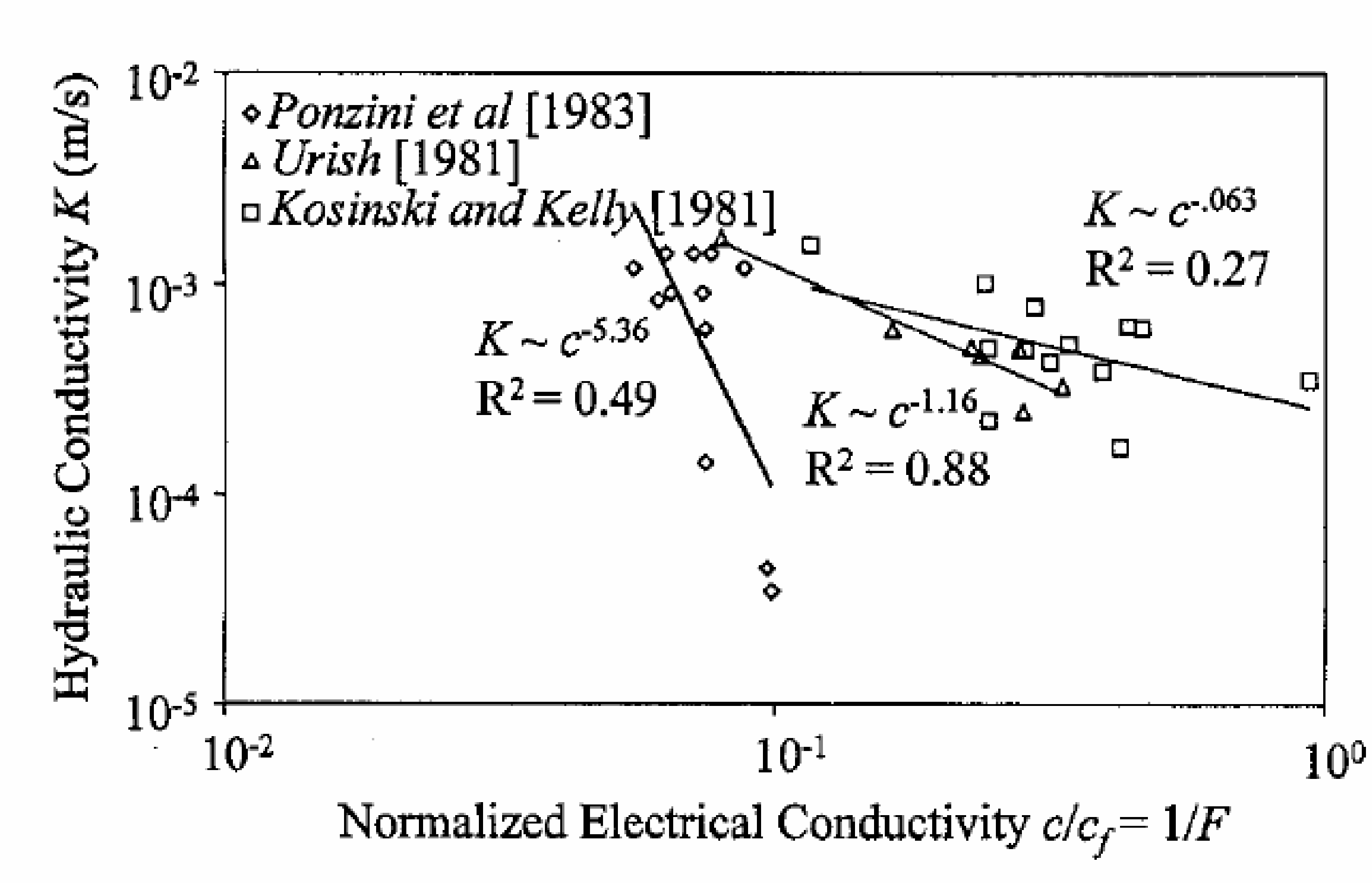
Similarly, we may expect to see a positive correlation between bulk conductivity and permeability



Purvance & Andrcevic (2000)

Permeability prediction – from conductivity/resistivity

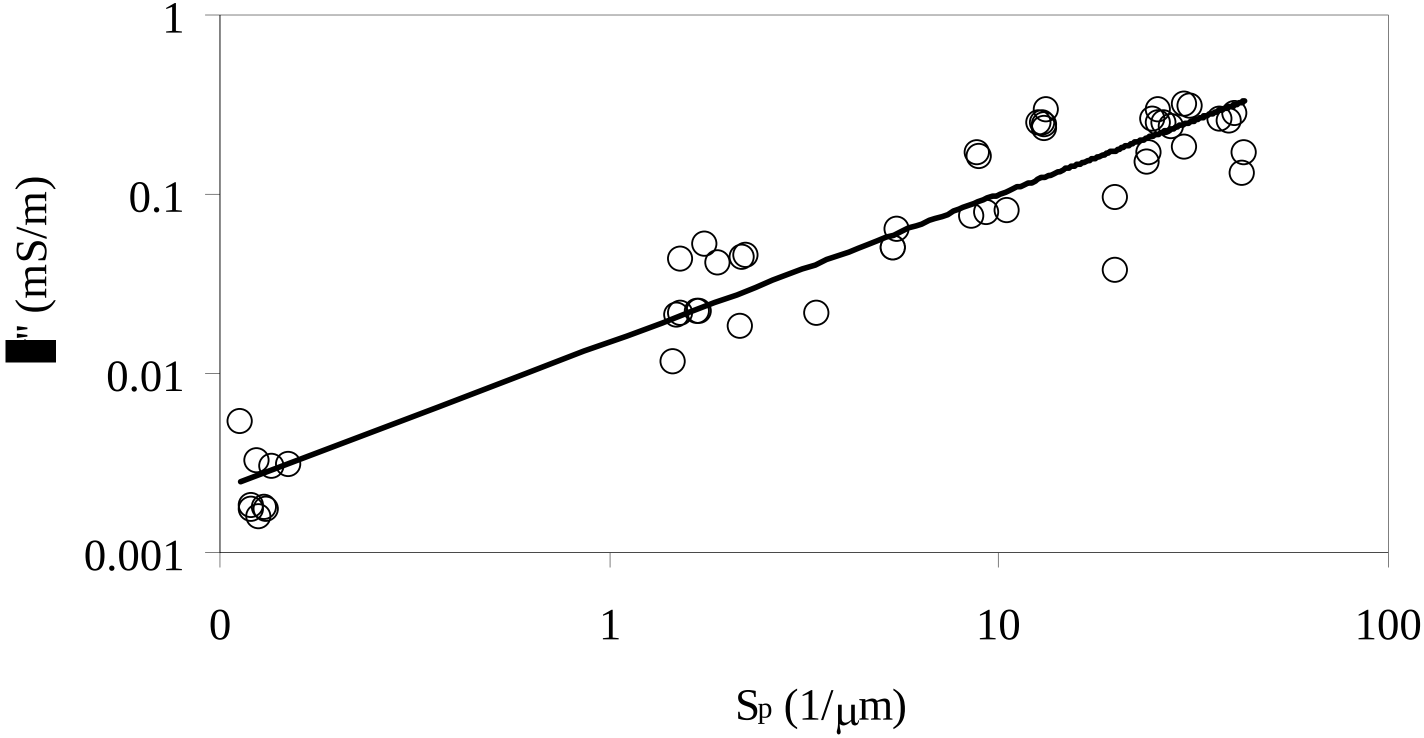
However, because of the surface conductivity effects the observed relationships may be weak or negative



Purvance & Andrcevic (2000)

Permeability prediction – using IP

IP may be able to account for the surface effects in our petrophysical models



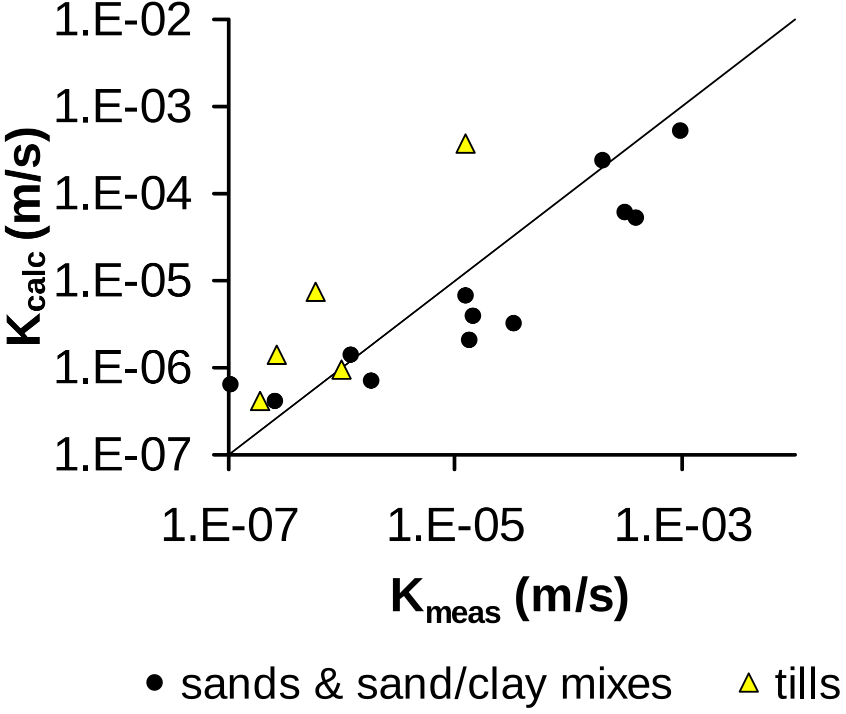
Specific surface area

Borner & Schon (1991)

Permeability prediction – using IP

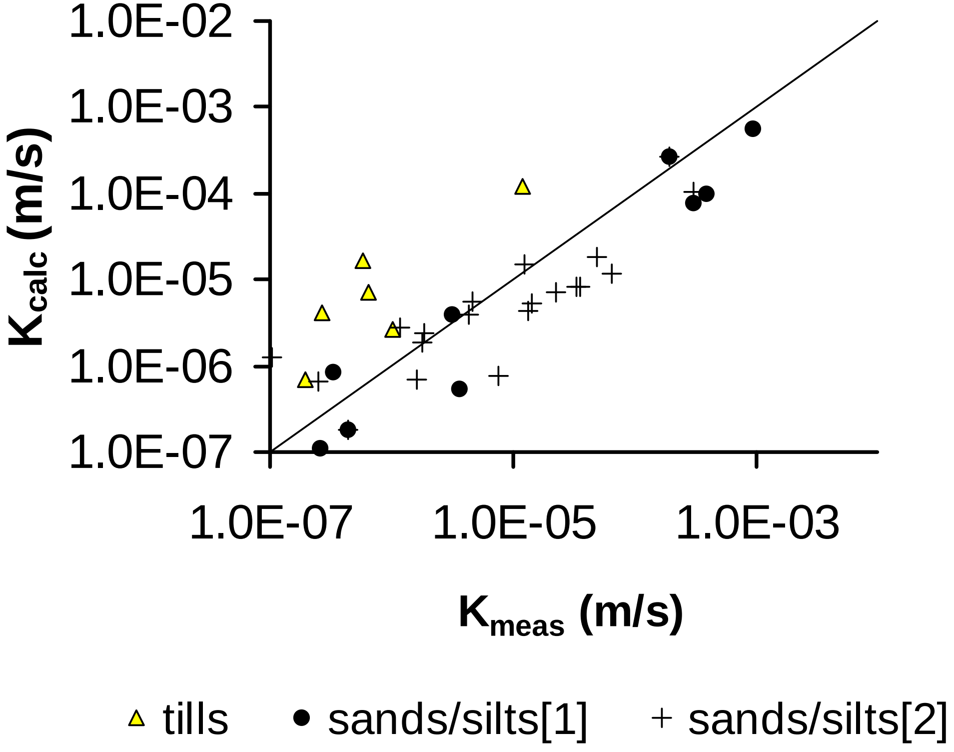
Borner & Schon (1991)
(modified Kozeny-Carmen model)

$$K_{calc} = \frac{a}{FS_{por[el]}^{c}}$$



Slater & Lesmes (2001)
Grain size model

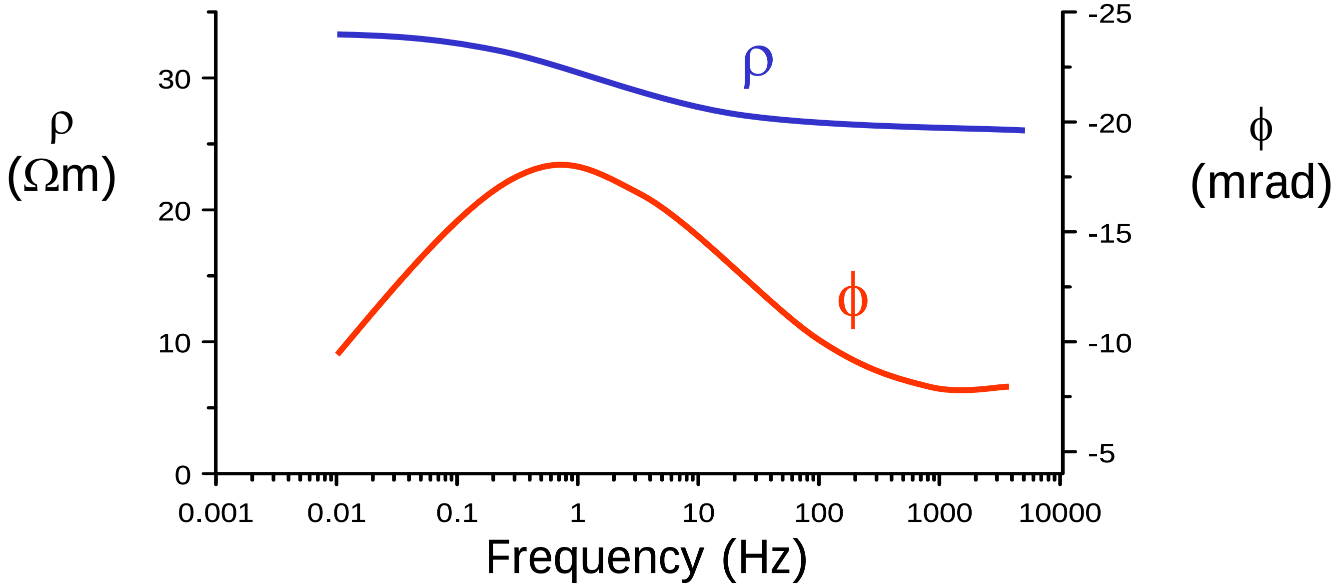
$$K_{calc} = 0.002(d_{10[EL]})^{d}$$



Permeability prediction – using IP

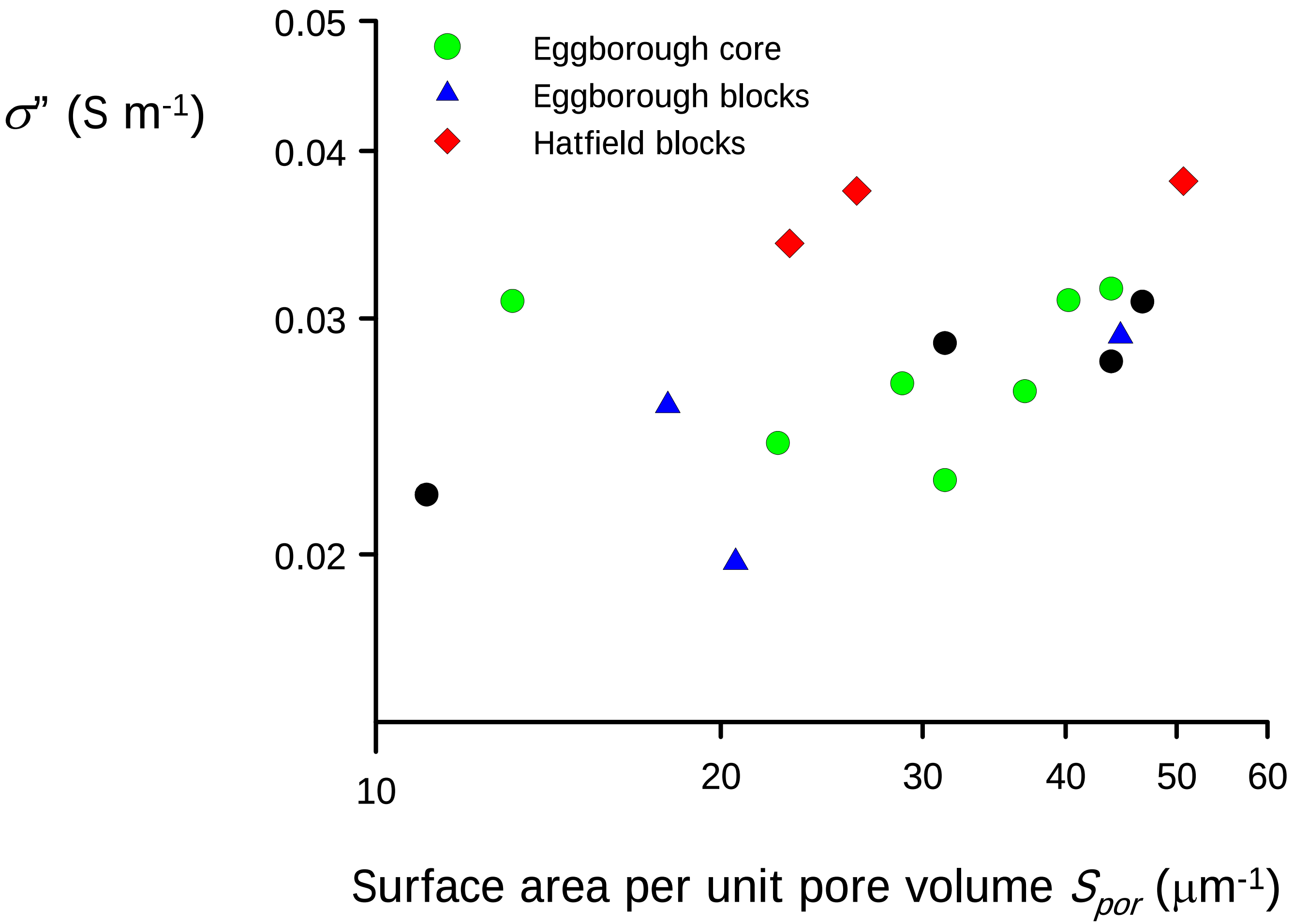
The model of Borner & Schon (1991) or Slater & Lesmes (2001) assume a frequency independent imaginary conductivity

In some cases this is not valid



Permeability prediction – using IP

If we take a single frequency in this case …



Binley *et al.* (2005)

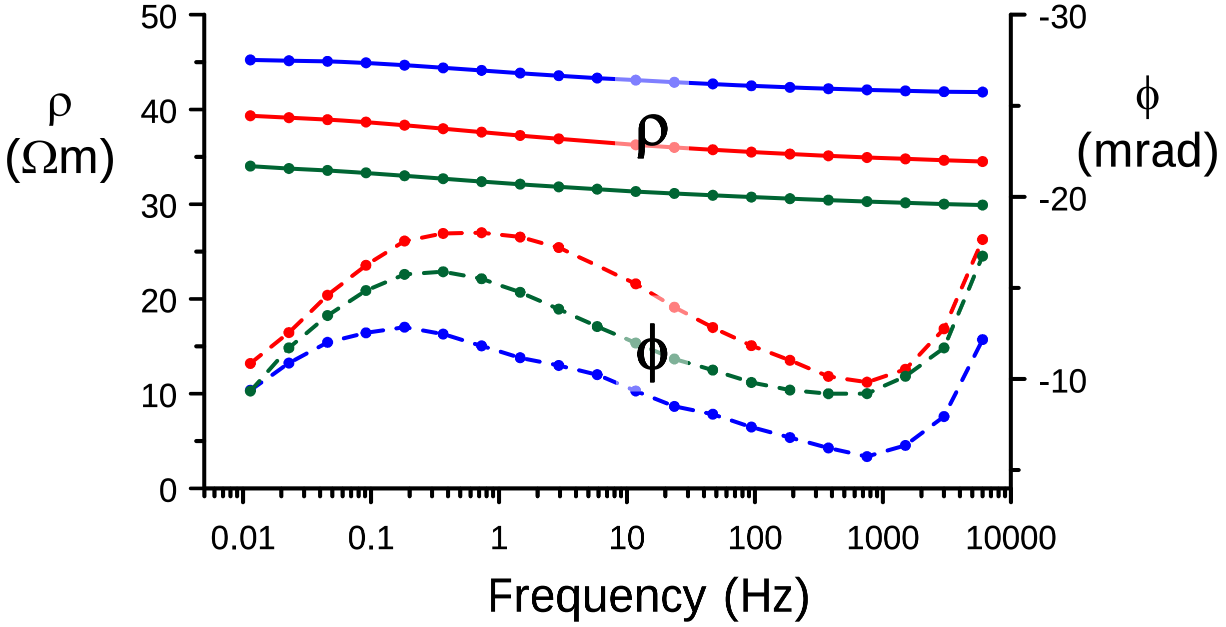
Permeability prediction – using Spectral IP

Can we use the spectral properties to estimate permeability?

VEC16-1
depth = 17.61 m

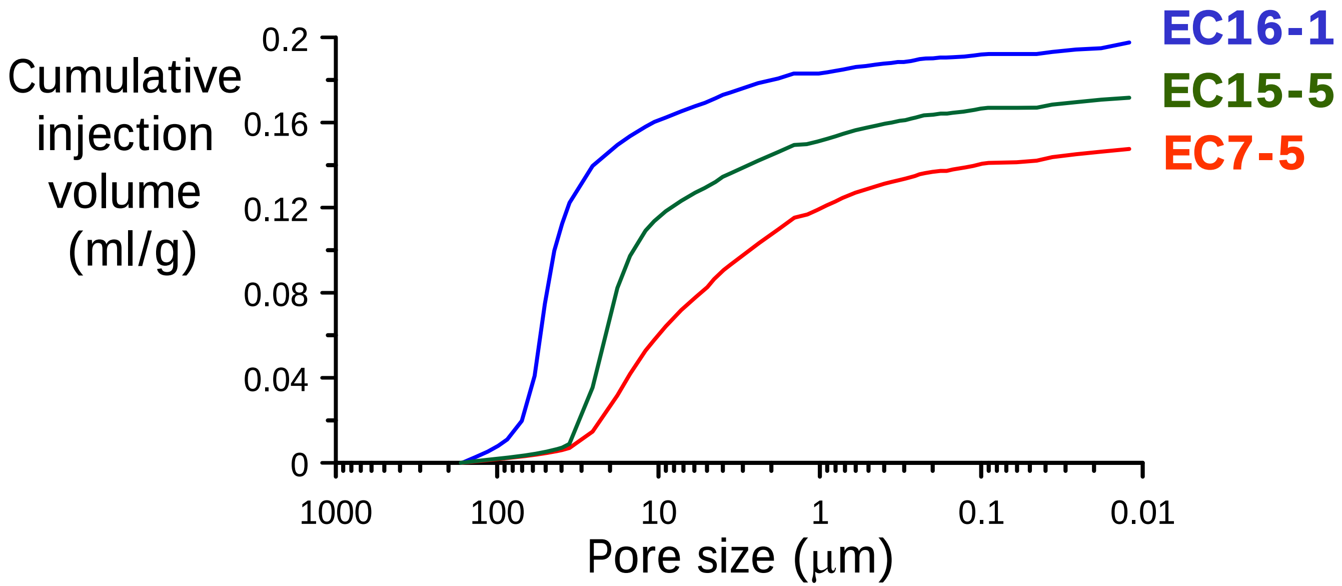
VEC15-5
depth = 16.07 m

VEC7-5
depth = 8.22 m



Binley *et al.* (2005)

Permeability prediction – using Spectral IP



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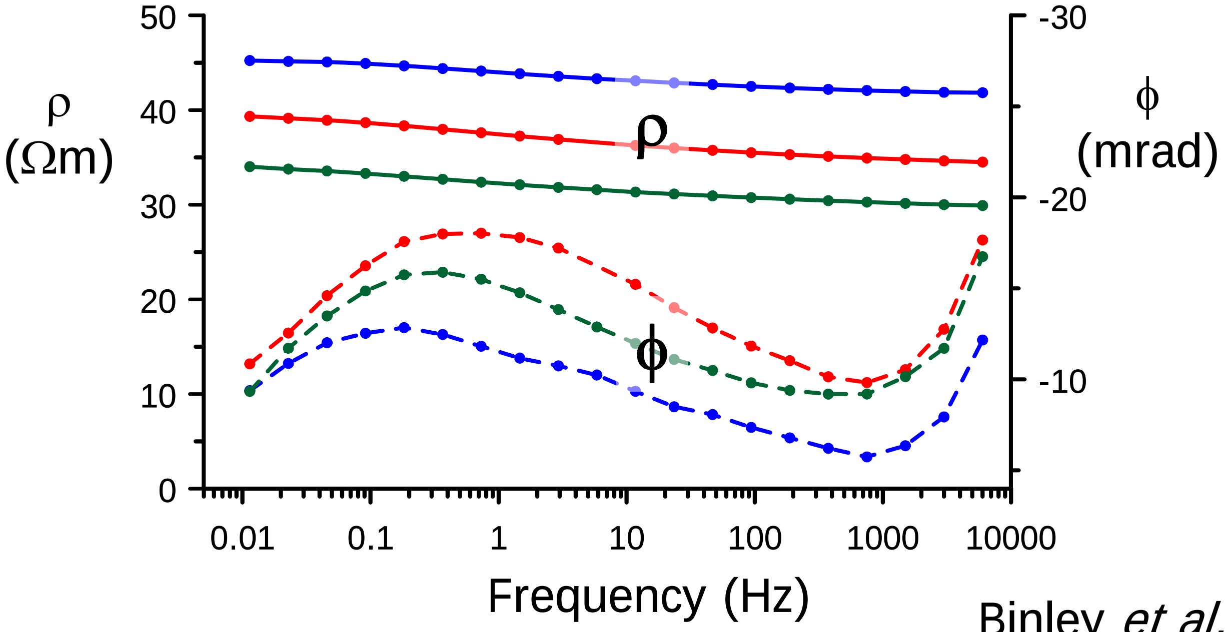
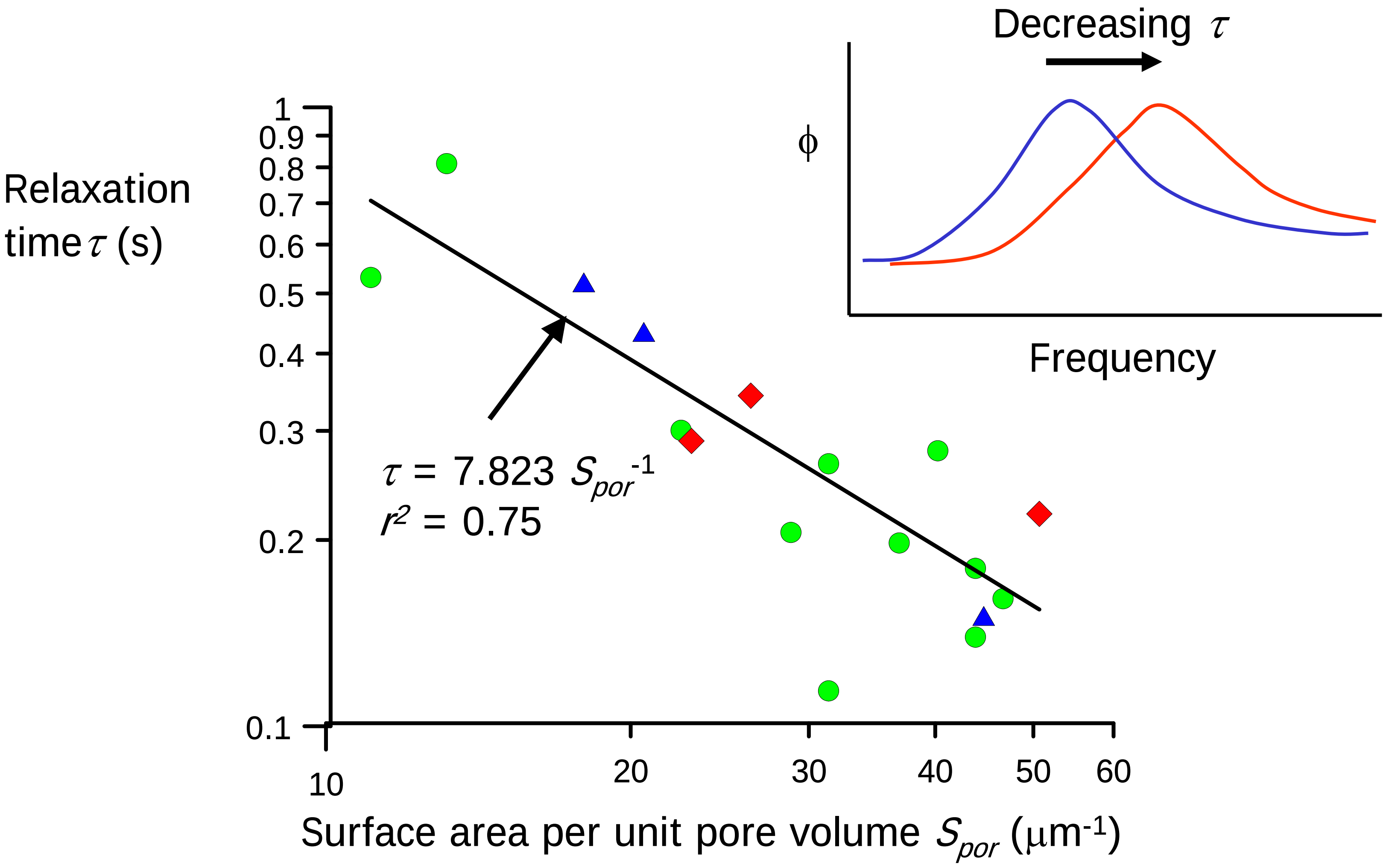


Binley *et al.* (2005)

Permeability prediction – using Spectral IP

The relaxation time is related to the surface area

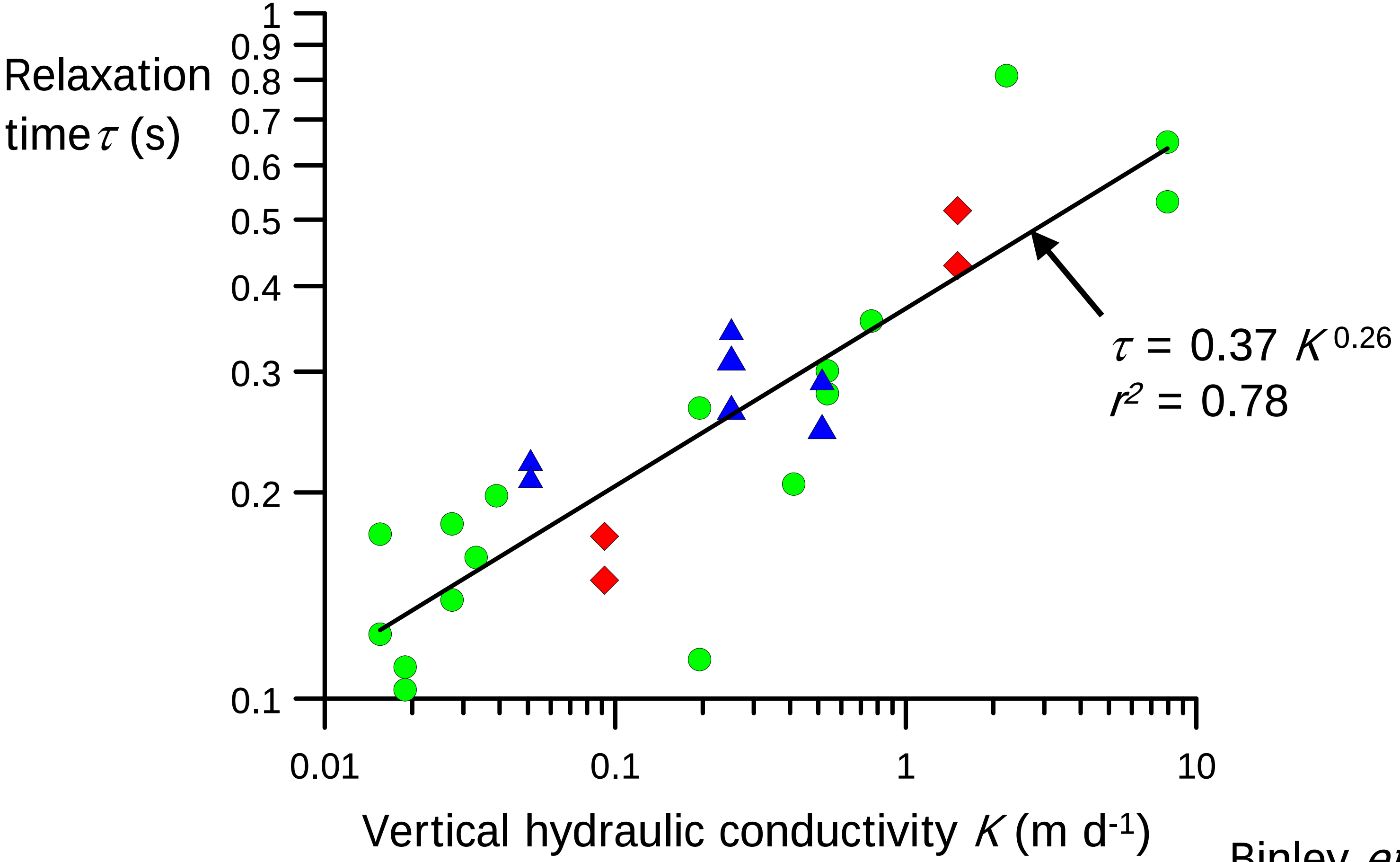


$\tau = 7.823\ S_{por}^{-1}$
$r^2 = 0.75$

Binley *et al.* (2005)

Permeability prediction – using Spectral IP

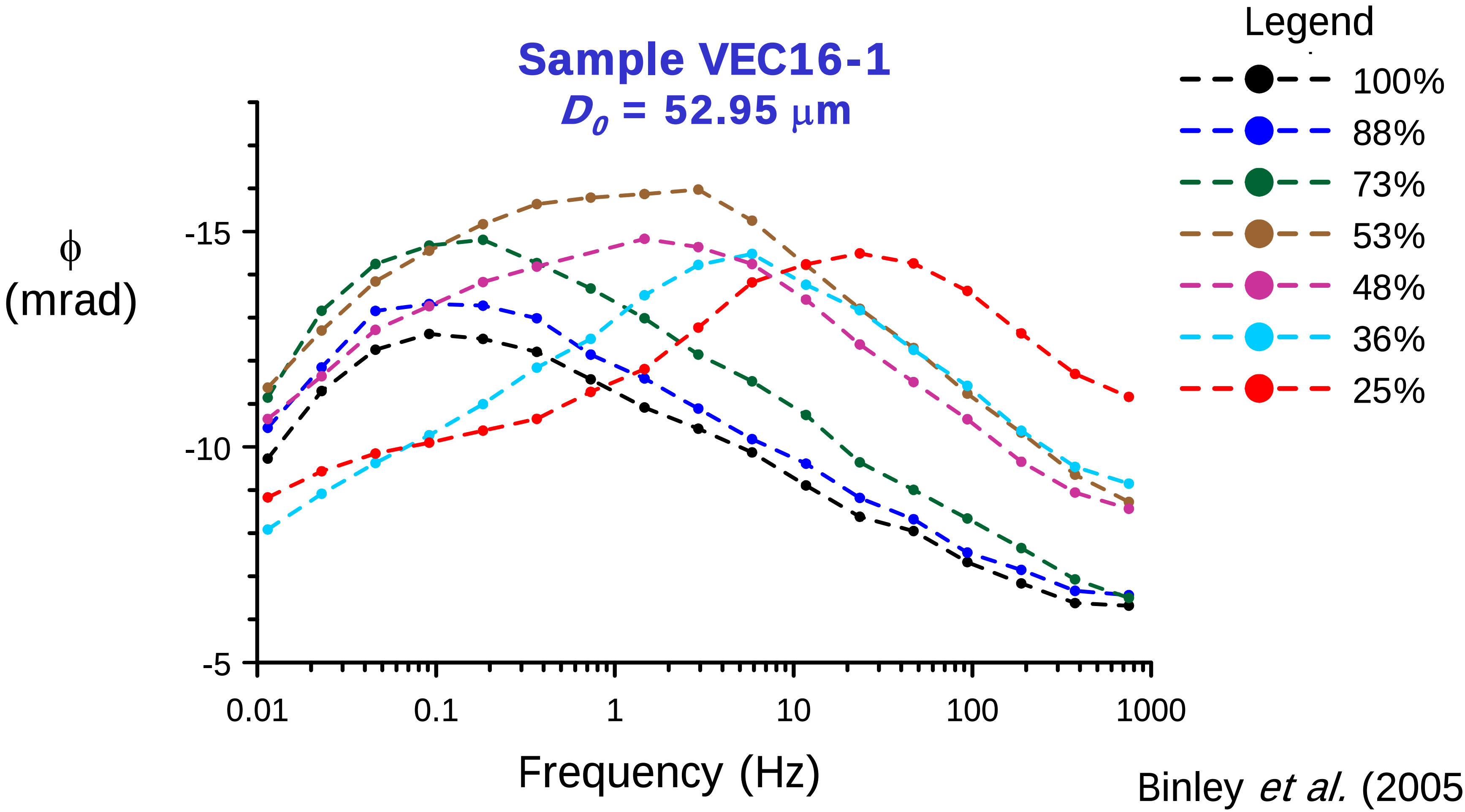
The relaxation time is then correlated to the permeability



Binley *et al.* (2005)

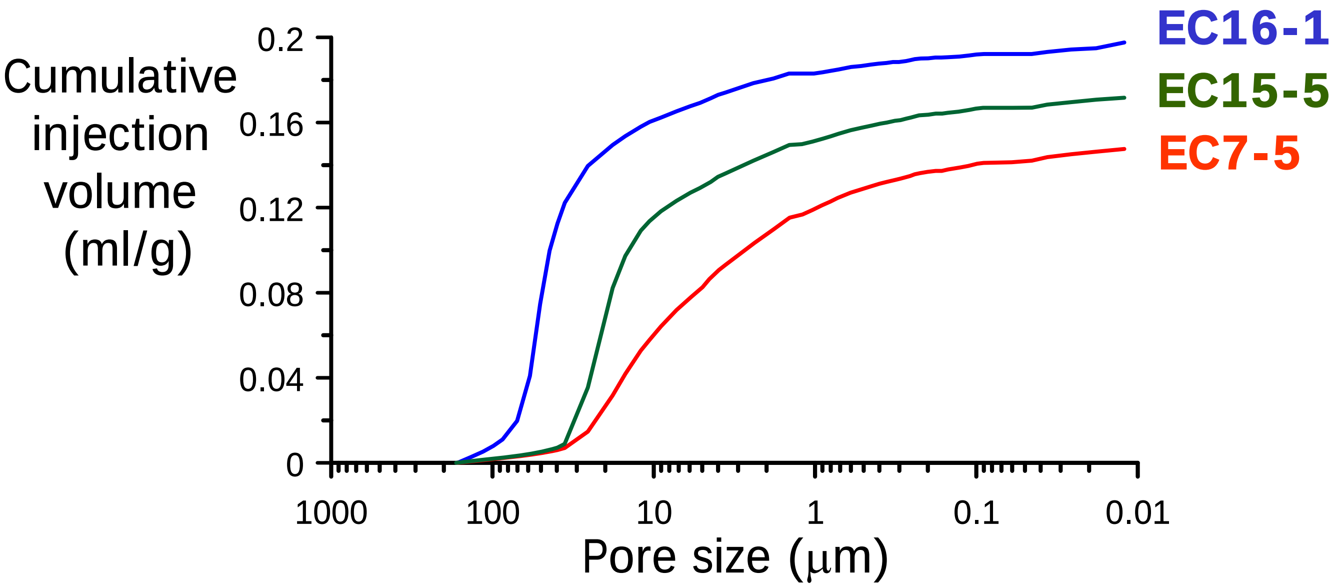
Permeability prediction – using Spectral IP

The spectra show sensitivity to saturation and so this must be taken into account in the vadose zone



Binley *et al.* (2005)

Unsaturated characteristics – using Spectral IP



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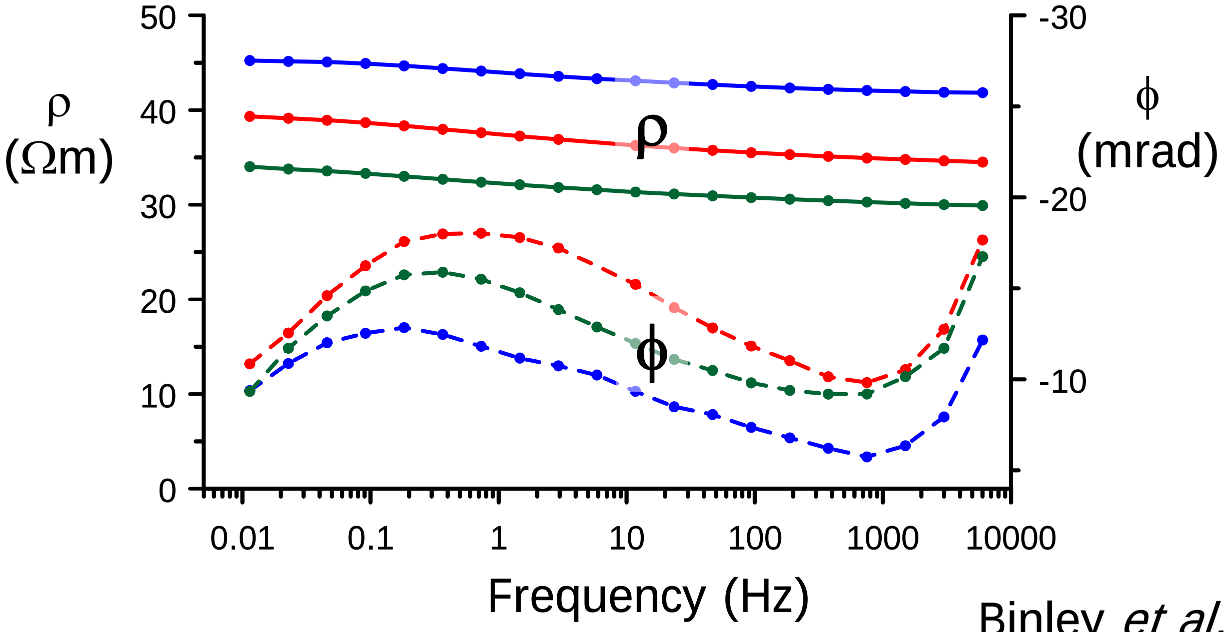
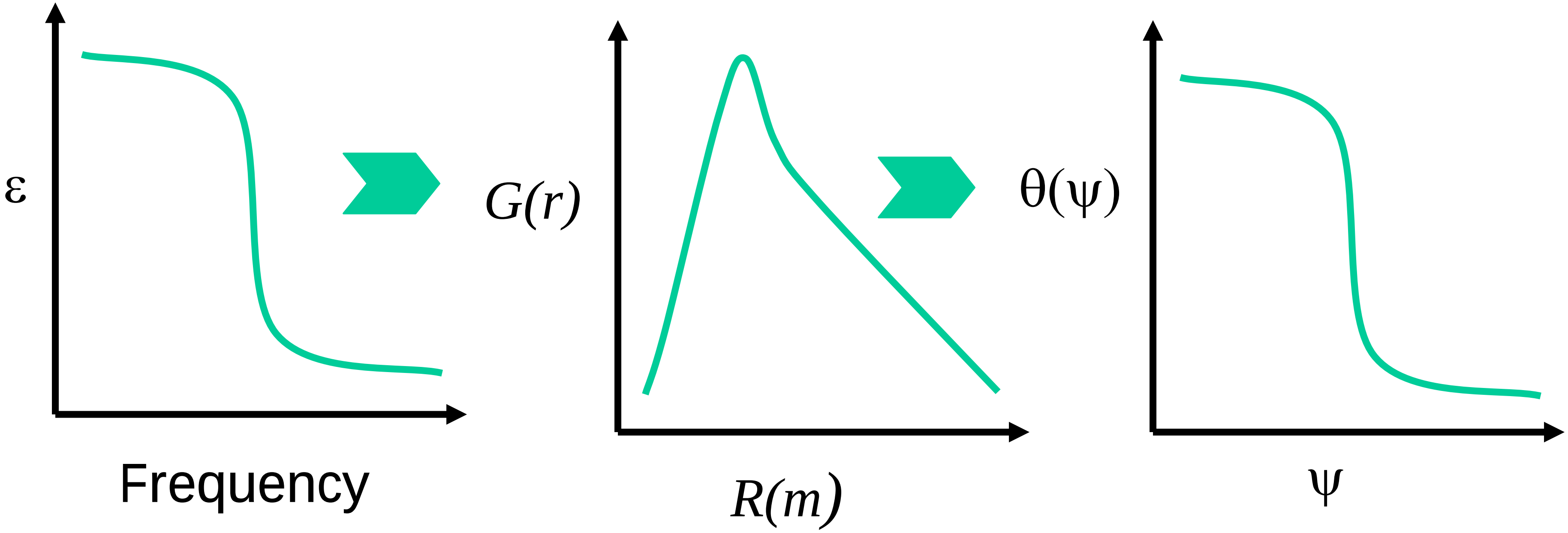


Binley *et al.* (2005)

Unsaturated characteristics – using Spectral IP

Can we estimate moisture retention curves using IP spectra?



Summary

Empirical and semi-empirical models are available to link hydrological and geophysical properties.

Some of these models may be site-specific.

We often need to account for sensitivity to various properties (moisture content, pore shape, clay content, etc).

Conductivity/resistivity – permeability relationships may be limited.

IP may help in accounting for surface controlled effects.

Spectral IP may offer greater value in constraining hydrological variables.